

• Percent Change of Dimensions

Dilation: Add the percent of increase to 100%.

Reduction: Subtract the percent of decrease from 100%.

Scale factor: To find the scale factor, convert the dilation or reduction percent to a decimal.

Example: The dimensions of a square are increased 20%. By what percent is the area increased?

1. The dilation is $100\% + 20\% = 120\%$.
2. The scale factor is $120\% = 1.2$.
3. Square the scale factor to determine the relationship between the two areas. (Remember: Area is the product of two dimensions.) $(1.2)^2 = 1.44$
4. Change into percent: $1.44 = 144\%$.

We can say that the area of the larger square is 144% of the area of the smaller square.

5. This means that the area is increased 44%. ($144\% - 100\% = 44\%$)

Practice:

1. Patrick reduced the size of a photograph by 30%.
 - a. What percent of the original size is the reduction? _____
 - b. What is the scale factor from the original to the reduction? _____
 - c. By what percent was the area of the photograph reduced? _____
2. A square is dilated 160%.
 - a. What is the scale factor of the dilation? _____
 - b. By what percent would the area increase? _____
3. Maggie's blanket uniformly shrunk 10% when it was put into the dryer.
 - a. What is the scale factor of the reduction? _____
 - b. The area of the blanket is what percent of its original area? _____
 - c. By what percent was the area of the blanket reduced? _____

• Multiple Unit Multipliers

Use two unit multipliers to convert two different units.

Example:

Convert 440 yd per minute to miles per hour:

$$\frac{440 \cancel{\text{yd}}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \cancel{\text{yd}}} = \frac{26,400 \text{ mi}}{1760 \text{ hr}} = 15 \text{ mph}$$

Use two unit multipliers to convert units of area.

Example:

Convert 288 square feet (ft²) to square yards (yd²):

$$288 \text{ ft}^2 \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \frac{288 \text{ yd}^2}{9} = 36 \text{ yd}^2$$

Use three unit multipliers to convert units of volume.

Example:

Convert 1107 cubic feet (ft³) to cubic yards (yd³):

$$1107 \text{ ft}^3 \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \frac{1107 \text{ yd}^3}{27} = 41 \text{ yd}^3$$

Practice:

1. Examine problems b–f.

a. For which problems will you need to use two unit multipliers?

Three unit multipliers?

b. 24 yards per minute to feet per second _____

c. 32 dollars per hour to cents per minute _____

d. 12 sq ft to sq in. _____

e. 1,000,000,000 cm³ to m³ _____

f. 3456 cubic inches to cubic ft _____

• Formulas for Sequences

$$3, 6, 9, 12, \dots \quad \text{Formula: } a_n = 3n$$

$$2, 4, 8, 16, 32, \dots \quad \text{Formula: } a_n = 2^n$$

$$3, 5, 7, 9, \dots \quad \text{Formula: } a_n = 2n + 1$$

To find a formula for a sequence, relate each term (a) with the number of the term (n .)

n	1	2	3	4
a	3	5	7	9

To find a term, double the number of the term and then add 1.

Practice:

1. $1, 4, 9, 16, 25, \dots$

a. What is the formula for the above number sequence?

b. What is the next number? _____

2. $2, 4, 6, 8, \dots$

a. What is the formula for the above number sequence?

b. Find the 10th term. _____

3. The terms of the following sequence are generated with the formula $a_n = n^n$. Find the next number in the sequence.

$$1, 4, 27, 256, \dots$$

4. The terms of the following sequence are generated with the formula $a_n = 2n - 1$.

$$1, 3, 5, 7, 9, \dots$$

What is the 12th number in the sequence? _____

Name _____

Math Course 3, Lesson 74

• Simplifying Square Roots

The Product Property of Square Roots

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

We can simplify square roots by removing perfect-square factors from the radical. We show two ways to simplify $\sqrt{12}$:

First Method:

Find the prime factors.

$$\begin{aligned}\sqrt{12} &= \sqrt{2 \cdot 2 \cdot 3} \\ &= \sqrt{2 \cdot 2} \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

Second Method:

Find the perfect square factor.

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \cdot 3} \\ &= \sqrt{4} \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

When using prime factors to simplify, look for pairs of identical factors. Each pair is a perfect square.

Example: Simplify $\sqrt{600}$.

FIRST METHOD:

Step 1: Factor 600.

$$\sqrt{600} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}$$

Step 2: Group pairs of identical factors.

$$\sqrt{2 \cdot 2} \cdot \sqrt{5 \cdot 5} \cdot \sqrt{2 \cdot 3}$$

Step 3: Simplify perfect squares.

$$2 \cdot 5 \cdot \sqrt{2 \cdot 3}$$

Step 4: Multiply.

$$10\sqrt{6}$$

SECOND METHOD:

$$\sqrt{600} = \sqrt{100 \cdot 6}$$

$$= \sqrt{100} \sqrt{6}$$

$$= 10\sqrt{6}$$

Practice:

Simplify if possible.

1. $\sqrt{140}$ _____

2. $\sqrt{80}$ _____

3. $\sqrt{500}$ _____

4. $\sqrt{1372}$ _____

5. $\sqrt{480}$ _____

6. $\sqrt{30}$ _____

• **Area of a Trapezoid**

Formula to Find the Area of a Trapezoid

$$A = \frac{1}{2}(b_1 + b_2) \cdot h$$

We read this formula: Area equals $\frac{1}{2}$ times the sum of the bases times the height.

The formula means: Multiply the average of the bases times the height.

Example: Find the area of the trapezoid.

Step 1: Add base 1 and base 2.

$$7 + 9 = 16$$

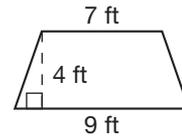
Step 2: Multiply the sum by $\frac{1}{2}$ or divide by 2.

The average of the bases is 8 ft. $\frac{1}{2}(16) = 8$

Step 3: Multiply the average of the bases by the height.

$$8(4) = 32$$

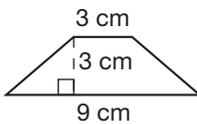
The area of the trapezoid is 32 square feet.



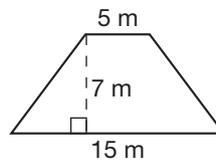
Practice:

Find the average of the bases and the area of each trapezoid.

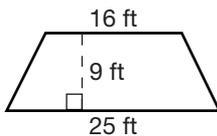
1.



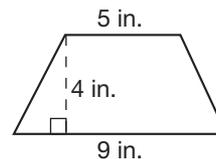
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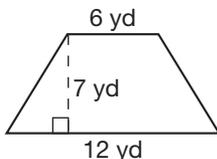
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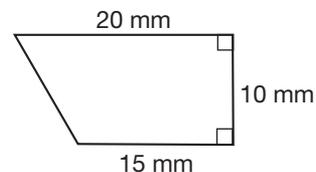
4.



5.



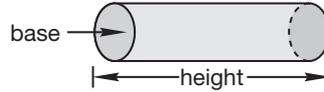
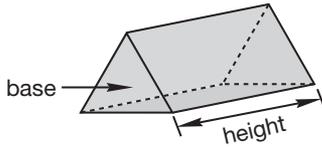
6.



Name _____

• Volumes of Prisms and Cylinders

- To find the volume of a prism or a cylinder perform these two steps:
 1. Find the area of the base.
 2. Multiply the area of the base times the height.



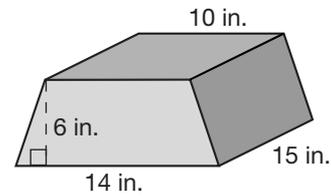
Practice:

1. Find the volume of a cylinder with a radius 5 ft and a height 10 ft.

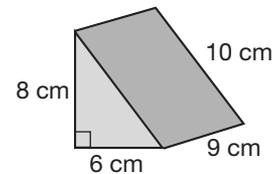
(Use 3.14 for π .) _____

2. Find the volume of a rectangular prism with length 6 in., height 4 in., and width 4 in. _____

3. Find the volume of this trapezoidal prism.



4. Find the volume of this triangular prism.

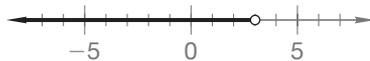


• Inequalities with Negative Coefficients

- Recall that we solve an inequality the way we solve an equation. However, if we multiply or divide by a negative number, we reverse the direction of the inequality.
- We can graph on a number line all the numbers that make the inequality true.

Example: Solve and graph: $-6(x - 3) > 6(x - 3)$

Step	Justification
$-6(x - 3) > 6(x - 3)$	Given equality
$-6x + 18 > 6x - 18$	Distributive Property
$-12x + 18 > -18$	Subtracted $6x$ from both sides
$-12x > -36$	Subtracted 18 from both sides
$x < 3$	Divided both sides by -12 and reversed the comparison symbol.



Practice:

Solve. Then graph the set of solutions.

1. $2x - 5x + 4 \leq 10$ _____

2. $4(x - 1) > 8$ _____

3. $11 - 2x \geq 3x + 16$ _____

4. $2(x - 2) \geq 5(x + 1)$ _____

5. $4(x + 2) < 5x - 1$ _____

• Products of Square Roots**Property of Square Roots**

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

This property means that square roots can be factored.

This property also means square roots can be multiplied.

This property can help you solve problems with square roots.

$$\begin{aligned} \text{Simplify: } & \sqrt{5} \cdot \sqrt{5} \\ & \sqrt{5} \cdot \sqrt{5} = \sqrt{25} \\ & \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} \text{Simplify: } & \sqrt{8} \cdot \sqrt{18} \\ & \sqrt{8} \cdot \sqrt{18} = \sqrt{144} \\ & \sqrt{144} = 12 \end{aligned}$$

$$\begin{aligned} \text{Simplify: } & \sqrt{10} \cdot \sqrt{14} \\ & \sqrt{10} \cdot \sqrt{14} = \sqrt{140} \\ & \sqrt{140} = \sqrt{2 \cdot 2 \cdot 5 \cdot 7} \\ & \sqrt{140} = \sqrt{2^2 \cdot 5 \cdot 7} \\ & \sqrt{140} = 2\sqrt{35} \end{aligned}$$

Practice:

1. $\sqrt{8}\sqrt{2}$ _____

2. $\sqrt{12}\sqrt{3}$ _____

3. $\sqrt{27} \cdot \sqrt{49}$ _____

4. $\sqrt{3} \cdot \sqrt{6}$ _____

5. $\sqrt{3} \cdot \sqrt{3}$ _____

6. $\sqrt{5} \cdot \sqrt{50}$ _____

• Transforming Formulas

Standard formulas are expressed with one variable isolated. You can transform, or rearrange, formulas before you solve a problem when you want to isolate a different variable.

Example: The formula for distance can be transformed to solve the equation to find the time:

Step	Justification
$d = rt$	Distance formula
$\frac{d}{r} = \frac{rt}{r}$	Divided both sides by r
$\frac{d}{r} = t$	Simplified
$t = \frac{d}{r}$	Symmetric property

Example: Solve for x : $w = x + b$

$w = x + b$	Equation
$w - b = x$	Subtracted b from both sides
$x = w - b$	Symmetric property of equality

Practice:

- Solve $A = lw$ for width. _____
- Solve $c = \pi d$ for the diameter. _____
- Solve $c^2 = a^2 + b^2$ for a . _____
- Transform this formula to solve for c . $a = bc$ _____
- Solve $d = rt$ for time. Then use your transformed formula to solve the following problem. Jan traveled a distance of 90 miles at the rate of 40 mph. How long did it take her to travel the 90 miles?
