Name ____________________

• Translating Expressions into Equations

An essential skill of mathematics is the ability to translate language, situations, and relationships into mathematical form (equations).

Examples of Translations

Phrase	Translation
twice a number	$2n$
five more than a number	$x + 5$
three less than a number	$a - 3$
half a number	$\frac{1}{2}h$ or $\frac{h}{2}$
the product of a number and seven	$7b$
seventeen is five more than twice a number	$17 = 2n + 5$

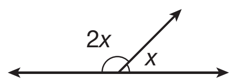
Example: If six less than three times a number is 15, what is the number?

$3x - 6 = 15$ equation

$3x = 21$ added 6 to both sides

$x = 7$ divided both sides by 3

Example: The angles marked x and 2x are supplementary. What is the measure of the larger angle?

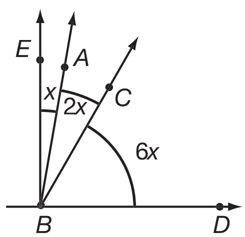


$2x + x = 180° \longrightarrow 3x = 180° \longrightarrow x = 60° \longrightarrow 2x = 120°$

The larger angle measures 120°.

Practice:

Use the figure below for 1–3. Write an equation to find the measure of each angle.



1. m∠*ABC* ________ **2.** m∠*EBA* ________ **3.** m∠*CBD* ________

4. Eleven less than the product of five and twelve is half of a number.

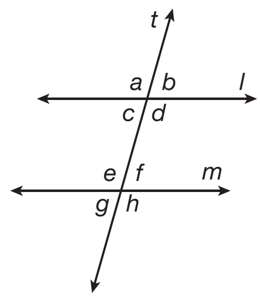
What is the number? ________

5. Four less than double what number is 50? ________

Name ______________________________

• Transversals
• Simplifying Equations

When a **transversal,** *t*, intersects two parallel lines, *l* and *m*, *corresponding* angles and *alternate* angles are formed.



- Corresponding angles ($\angle a$ and $\angle e$; $\angle c$ and $\angle g$; $\angle b$ and $\angle f$; $\angle d$ and $\angle h$) have the same measure.

  Corresponding angles lie on same side of a transversal in matching positions along parallel lines.
- Alternate *exterior* angles ($\angle a$ and $\angle h$; $\angle b$ and $\angle g$) have the same measure.

  Alternate exterior angles lie on opposite sides of a transversal and outside parallel lines.
- Alternate *interior* angles ($\angle d$ and $\angle e$; $\angle c$ and $\angle f$) have the same measure.

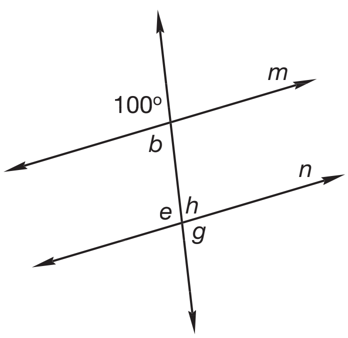
  Alternate interior angles lie on opposite sides of a transversal and inside parallel lines.
- To **simplify equations:**
  1. Collect like terms on each side of the equals sign.
  2. Move all variable terms to one side of the equation.
  3. Move all constant terms to the other side of the equation.
  4. Solve for the variable.

Example:

$4x + x - 10 = 2(x + 1)$	equation
$4x + x - 10 = 2x + 2$	distributive property
$5x - 10 = 2x + 2$	collected like terms
$3x - 10 = 2$	subtracted $2x$ from both sides
$3x = 12$	added 10 to both sides
$x = 4$	divided both sides by 3

Practice:

Use the figure below for 1–4. Find the measure of each angle.
Lines *m* and *n* are parallel.



1. $\angle b =$ ________ **2.** $\angle h =$ ________

3. $\angle e =$ ________ **4.** $\angle g =$ ________

Solve 5 and 6.

5. $2y + 15 - y = 4(y - 3)$ ________

6. $6a + 5 + a = 5(a + 3)$ ________

Name ______________________________

• Powers of Negative Numbers
• Dividing Terms
• Square Roots of Monomials

- To multiply signed numbers:
  1. Multiply the numbers, disregarding the signs.
  2. Count the **negative signs.**
     An **even** number of negative signs gives a **positive** product.
     An **odd** number of negative signs gives a **negative** product.
     Signs of positive factors do not affect the sign of the product.
  3. Place the correct sign on the product.

  Example: Find the product of $(-3)(-4)(+5)(-2)(+3)$.
  Three negative signs (odd number) $=$ a negative product
  $(-3)(-4)(+5)(-2)(+3) = -360$

- To divide terms with variables:
  1. Factor terms in numerator and in denominator.
  2. Cancel matching factors.
  3. Regroup remaining factors.

  Example: $\dfrac{10a^3bc^2}{8ab^2c} = \dfrac{\overset{1}{\cancel{2}} \cdot 5 \cdot \overset{1}{\cancel{a}} \cdot a \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{c}} \cdot c}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot \underset{1}{\cancel{c}}} = \dfrac{5a^2c}{4b}$

- A monomial is a **perfect square** if its prime factorization can be separated into two identical groups of factors.

  Example: What is the square root of $25x^2$?
  $25x^2 = 5 \cdot 5 \cdot x \cdot x$
  $= 5x \cdot 5x$
  So $\sqrt{25x^2} = 5x$

- Shortcut: Find the square root of the constant and half of each exponent.

  Example: Simplify $\sqrt{49a^2b^6}$
  $\sqrt{49a^2b^6} = 7ab^3$

Practice:

Simplify 1–6.

1. $(-2)^3 - (-4)^2$ __________

2. $\dfrac{(3x)(5x^2y)}{3x^3y}$ __________

3. $6^2 \cdot (-2)^3$ __________

4. $\sqrt{64x^6y^8}$ __________

5. $\sqrt{16a^6b^2c^4}$ __________

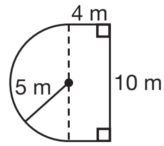
6. $\dfrac{26w^3yz^4}{13wyz}$ __________

Name ________________________________

• Semicircles, Arcs, and Sectors

- A **semicircle** is half a circle.

  Perimeter of semicircle $= \frac{\pi d}{2}$ *Area* of semicircle $= \frac{\pi r^2}{2}$

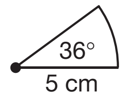
  

  Example: Find the perimeter of this figure:

  Perimeter of semicircle $= \frac{\pi d}{2} \approx \frac{3.14(10\text{ m})}{2} \approx 15.7\text{ m}$

  Perimeter of figure $\approx 15.7\text{ m} + 4\text{ m} + 10\text{ m} + 4\text{ m} \approx 33.7\text{ m}$

- An **arc** is a part of the circumference of a circle.
- To find the length of an arc:
  1. Find the fraction of the circle that the arc represents.
  2. Find the circumference of the whole circle. (Remember $C = \pi d$.)
  3. Multiply the circumference by the fraction.

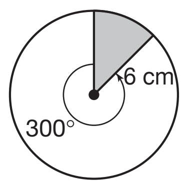
  Example: Find the length of this arc.

  Fraction of circle $= \frac{36°}{360°} = \frac{1}{10}$

  Circumference of circle $= \pi d \approx (3.14)(10\text{ cm}) \approx 31.4\text{ cm}$

  Arc length $\approx \frac{1}{10} \cdot 31.4\text{ cm} \approx 3.14\text{ cm}$

- A **sector** of a circle is a portion of a circle bordered by part of the circumference (an arc) and two radii.
- To find the area of a sector:
  1. Find the fraction of the circle.
  2. Find the area of the whole circle. ($A = \pi r^2$)
  3. Multiply the area by the fraction.

  Example: Find the area of the shaded sector of this circle. The whole circle is 360°, so the angle of the shaded sector is 60°. $(360° - 300° = 60°)$

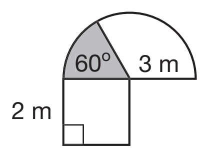
  Fraction of circle $= \frac{60°}{360°} = \frac{1}{6}$

  Area of circle $= \pi r^2 \approx 3.14(6\text{ cm})^2 \approx 113.04\text{ cm}^2$

  Area of shaded sector $\approx \frac{1}{6} \cdot 113.04\text{ cm}^2 \approx 18.84\text{ cm}^2$

Practice:

Use the figure on the right for 1–3. Use 3.14 for π.



1. Find the perimeter of the figure. __________

2. Find the area of the figure. __________

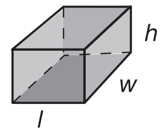
3. Find the perimeter and the area of the shaded sector.

____

Name ______________________________

• Surface Area of a Right Solid
• Surface Area of a Sphere

- To find the **surface area of a right solid:**
  1. Find the area of each face.
  2. Add the areas.

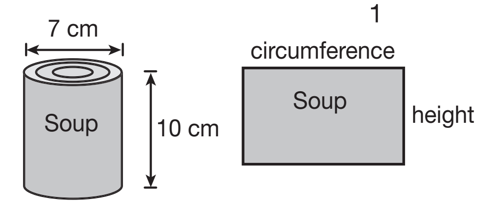


$$A = 2lw + 2lh + 2wh$$

To find the surface area of the side of a cylinder, multiply the circumference by the height.

Example: What is the area of the label on this soup can?

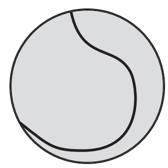
$$\text{Area} = \pi d \cdot \text{height} \approx \frac{22}{\cancel{7}_1}(\cancel{7}^1 \text{ cm}) \cdot 10 \text{ cm} \approx 220 \text{ cm}^2$$



Use $\frac{22}{7}$ for π.

- To find the **surface area of a sphere,** use this formula:
  Surface area of a sphere $= 4\pi r^2$

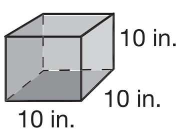
Example: A tennis ball has a diameter of about 6 cm. Find the surface area of the tennis ball to the nearest square centimeter. Use 3.14 for π.



Surface area $= 4\pi r^2 \approx 4(3.14)(3 \text{ cm})^2 \approx (12.56)(9 \text{ cm}^2) \approx 113.04 \text{ cm}^2$
Round 113.04 cm^2 to 113 cm^2.

Practice:

Find the surface area of each solid. Use 3.14 for π.

1. 

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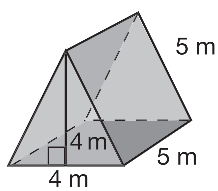
2. 

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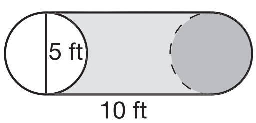
3. 

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- **Solving Literal Equations**
- **Transforming Formulas**
- **More on Roots**

- **Literal equations** use letters instead of numbers.
  Solve literal equations like equations that have numbers.
  Isolate the variable to be solved.

Example: Solve $ax = b$ for x.

$ax = b$ equation

$\frac{\overset{1}{\cancel{a}}x}{\underset{1}{\cancel{a}}} = \frac{b}{a}$ divided by a

$x = \frac{b}{a}$ simplified (isolated x)

- Formulas are literal equations used to solve certain kinds of problems.
  Formulas can be rearranged to solve for one of the variables.

Example: Solve $A = lw$ for w.

$A = lw$ equation

$\frac{A}{l} = \frac{\overset{1}{\cancel{l}}w}{\underset{1}{\cancel{l}}}$ divided by l

$w = \frac{A}{l}$ simplified (isolated w)

- A perfect square has both positive and negative square roots.

Example: $6 \cdot 6 = 36$ and $(-6)(-6) = 36$
The equation $x^2 = 36$ has two solutions, $x = 6$ and $x = -6$.
Note that $\sqrt{x^2}$ always indicates the positive square root of x.

- A radical sign can indicate other roots besides square roots.

Example: Simplify $\sqrt[3]{125}$.
$\sqrt[3]{125}$ means the cube root of 125.
$125 = 5 \cdot 5 \cdot 5$ so $\sqrt[3]{25} = 5$

Example: Simplify $\sqrt[3]{-216}$.
$-216 = (-6)(-6)(-6) \longrightarrow \sqrt[3]{-216} = -6$

Practice:

1. Solve the equation $a = bc$ for c. ________
2. Solve the equation $R = \frac{ht}{2}$ for h. ________
3. What are the two solutions for the equation $y^2 = 169$? ________
4. Simplify $\sqrt{\frac{169}{13}}$. ________
5. Simplify $\sqrt[3]{-27}$. ________

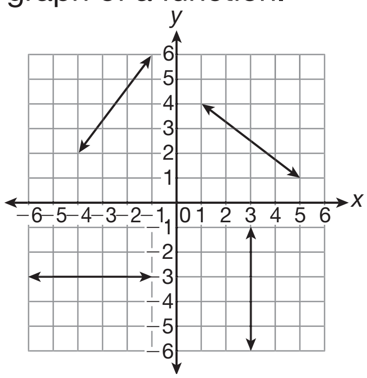
Name ______________________________

• Slope

- The slope is the slant of the graph of a function.

Upward slope is positive.

Downward slope is negative.



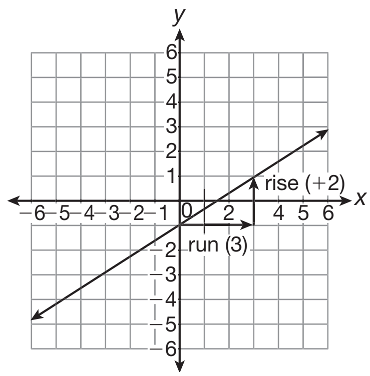
Vertical line—slope is zero.

Vertical line—slope cannot be determined.

- The slope of a line is the ratio of its rise to its run. The formula is slope $= \frac{\text{rise}}{\text{run}}$.

Example: Find the slope of the graphed line.

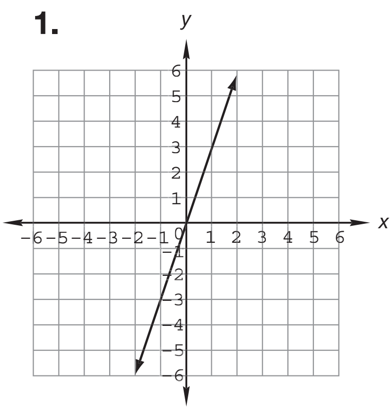
1. Locate two points on the line. Use (0, −1) and (3, 1).
2. Start from the point on the left (0, −1), and draw the horizontal leg *to the right.*
3. Then draw the vertical leg up or down to the second point (3, 1).
4. Find the rise and the run. The rise is +2 and the run is 3.
5. The slope is $\frac{2}{3}$. (Since the line rises upward to the right, the slope is positive.)



Practice:

Find the slope of each line.

1.



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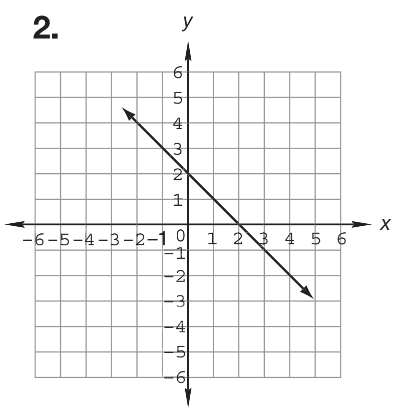

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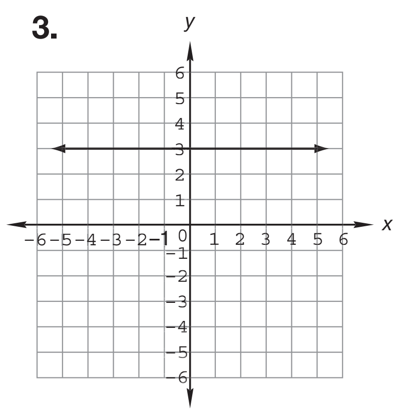

__________ __________ __________

Name ____________________

• Formulas and Substitution

- A **formula** is an equation that uses letters to tell about two or more unknown numbers.
- To use a formula:
  1. Substitute numbers that are known for the letters in the formula.
  2. Solve.

Example: Use the formula $d = rt$ to find t when d is 36 and r is 9.
This formula describes the relationship between distance (d), rate (r), and time (t).

$d = rt$	formula
$36 = 9t$	substituted
$t = 4$	divided by 9

Example: Use the formula $F = 1.8C + 32$ to find F when C is 37.
This formula is used to convert measurements of temperature from degrees Celsius to degrees Fahrenheit.

$F = 1.8C + 32$	formula
$F = 1.8(37) + 32$	substituted
$F = 66.6 + 32$	multiplied
$F = 98.6$	added

Thus, 37 degrees Celsius equals 98.6 degrees Fahrenheit.

Practice:

1. Use the formula $P = 2(l + w)$ to find w when P is 50 and l is 20.

____

2. Use the formula $P = ns$ to find s when P is 100 and n is 4.

____

3. Use the formula $C = \frac{5}{9}(F - 32)$ to find C when F is 41.

____

4. Use the formula $x = 4.07g$ to find g when x is 20.35.

____

Name ____________________

• Equations with Exponents

- When the variable of an equation has an exponent, the equation has up to that many solutions.

  If the variable has an exponent of 2, there are up to 2 solutions.
  If the variable has no exponent (it is understood to be 1), there is up to 1 solution.

- Some equations with exponents can be solved by isolating the variable.
  1. Add or subtract on both sides to isolate the variable *term.*
  2. Then multiply or divide on both sides to isolate the variable.
  3. Find the root (square root, cube root, etc.) of both sides.

Example: Solve $3x^2 - 1 = 47$.

$3x^2 = 48$ added 1 to both sides

$x^2 = 16$ divided both sides by 3

$x = 4, -4$ found square root of both sides

Example: Solve $2x^2 = 10$.

$x^2 = 5$ divided both sides by 2

$x = \sqrt{5}, -\sqrt{5}$ found square root of both sides

Since $\sqrt{5}$ is an irrational number, we leave it in radical form.

Notice that the negative of $\sqrt{5}$ is $-\sqrt{5}$ and not $\sqrt{-5}$.

Example: Solve $\frac{x}{3} = \frac{12}{x}$.

$x^2 = 36$ cross-multiplied

$x = 6, -6$ found square root of both sides

There are two solutions to the proportion, 6 and –6.

Practice:

Solve each equation.

1. $6y^2 = 42$ ____________

2. $2x^2 - 10 = 118$ ____________

3. $\frac{5}{t} = \frac{t}{5}$ ____________

4. $\frac{3x}{4} = \frac{6}{x}$ ____________

5. $7x^2 - 50 = 650$ ____________

6. $3y^2 = 243$ ____________

Name ______________________________

• Simple Interest and Compound Interest
• Successive Discounts

- **Principal** is the money you deposit in a bank.
- **Interest** is the money your money earns.
  Compound interest is interest earned on the principal and accumulated interest.
  Simple interest is earned on the principal only.

Compound Interest

principal	\$100.00
1st yr interest (6% of \$100.00)	6.00
total after one year	\$106.00
2nd yr interest (6% of \$106.00)	6.36
total after two years	\$112.36

Simple Interest

principal	\$100.00
1st yr interest	6.00
2nd yr interest	+ 6.00
total after two years	\$112.00

- A **successive discount** is a discount on an already discounted price.

Example: A \$400 appliance was reduced 25%. This sale price was then reduced 20% to its clearance price. What was the clearance price?

Sale Price

	Percent	Actual Count
Original	100	400
− Change	25	*D*
New (Sale)	75	*S*

$$\frac{100}{75} = \frac{400}{S}$$
$$100 \cdot S = 75 \cdot 400$$
$$100S = 30{,}000$$
$$S = \frac{30{,}0\not{0}\not{0}}{1\not{0}\not{0}}$$
$$S = \$300$$

Clearance Price

	Percent	Actual Count
Original (Sale)	100	300
− Change	20	*D*
New (Clearance)	80	*C*

$$\frac{100}{80} = \frac{400}{C}$$
$$100 \cdot C = 80 \cdot 300$$
$$100C = 24{,}000$$
$$C = \frac{24{,}0\not{0}\not{0}}{1\not{0}\not{0}}$$
$$C = \$240$$

Another way to solve: Sale price = 75% of the original price.
Clearance price = 80% of the sale price.
Clearance price = 80% of 75% of \$400
= 0.8 × 0.75 × \$400 = 0.6 × \$400 = \$240

Practice:

1. Maria put \$2000 into an account that paid 4% interest compounded annually.

   How much did she earn in 3 years? ________

2. A \$220 coat was reduced 20%. The final clearance price was 10% off the reduced price. What was the clearance price? ________

3. Jill's savings account pays 3% interest compounded annually.

   How much interest does Jill earn on \$500 in 2 years? ________