

Name _____

• **Prime and Composite Numbers**
 • **Prime Factorization**

- A **prime** number is a whole number greater than 1 that has exactly two factors, the number itself and 1.

Examples: 2—Factors of 2 are 1 and 2.
 3—Factors of 3 are 1 and 3.

- A **composite** number has more than two factors.

Examples: 4—Factors of 4 are 1, 2, and 4.
 6—Factors of 6 are 1, 2, 3, and 6.

- **Prime factorization** expresses a composite number as a product of its prime factors.

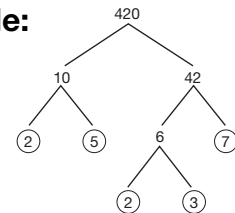
Examples: $4 = 2 \cdot 2$ $6 = 2 \cdot 3$ $9 = 3 \cdot 3$ $12 = 2 \cdot 2 \cdot 3$

- To factor a number using a **factor tree**:

1. Write any two factors of the given number.
2. Continue this process until each factor is a prime number.
3. Circle the prime factors.
4. Write the prime factors in order.

Notice that some factors will occur more than once.

Example:

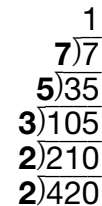


$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$

- To factor a number using **division by primes**:

1. Write the given number in a division box.
2. Begin dividing by a prime number that is a factor.
3. Divide the answer by a prime number that is a factor.
4. Repeat this process until the quotient is 1.
5. The divisors are the prime factors of the given number. Write the prime factors in order.

Example:



$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$

Practice:

1. List the composite numbers from 1 to 20? _____

Write the prime factorization of the following numbers.

2. 42 _____ 3. 24 _____ 4. 60 _____

5. Write the prime factorization of 64 and $\sqrt{64}$, using exponents.

• **Problems About a Fraction of a Group**

- To solve problems about a fraction of a group use diagrams.

Example: Debbie scored $\frac{2}{3}$ of her team's 36 points.
How many points did she score?

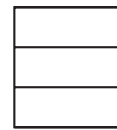
1. Draw a rectangle. This stands for the total.

36 points



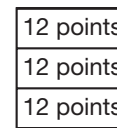
2. Divide the rectangle into the same number of parts as the denominator. The denominator of $\frac{2}{3}$ is 3, so divide the rectangle into 3 equal parts.

36 points



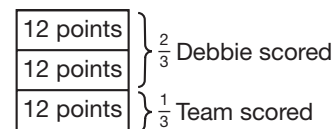
3. Divide the total by the denominator.
 $36 \div 3 = 12$
Write that answer in each part.

36 points



4. Bracket the parts into fractions.
Debbie scored 24 points.

36 points



Practice:

Draw a diagram of each statement. Then answer the questions that follow.

Fifty people saw the movie. Two fifths were children.

1. How many children saw the movie? _____
2. How many adults saw the movie? _____

50 people

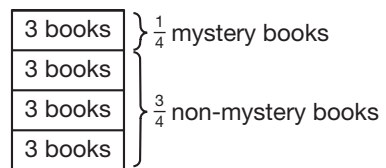


Twenty-five percent of the 12 books were mysteries.

3. What fraction of the books were not mysteries?

4. How many books were mysteries? _____

12 books



- **Subtracting Mixed Numbers with Regrouping**

- To subtract mixed numbers that require regrouping:
 1. **Borrow** 1 from the whole number and rename as a fraction.
 2. **Combine** the top fraction with the renamed 1.
 3. Then subtract.
 4. Reduce if possible.

Example:

$$\begin{array}{r}
 3\frac{1}{5} \\
 - 1\frac{2}{5} \\
 \hline
 \end{array}
 \longrightarrow
 2 + \frac{5}{5} + \frac{1}{5}
 \longrightarrow
 \begin{array}{r}
 2\frac{6}{5} \\
 - 1\frac{2}{5} \\
 \hline
 1\frac{4}{5}
 \end{array}$$

Practice:

Simplify 1–6.

1. $6 - 2\frac{5}{8}$ _____

2. $4\frac{1}{3} - 3\frac{2}{3}$ _____

3. $50\% - 12\frac{1}{2}\%$ _____

4. $25\frac{1}{6}\% - 5\frac{5}{6}\%$ _____

5. $27\frac{1}{7} - 13\frac{5}{7}$ _____

6. $100\% - 73\frac{3}{4}\%$ _____

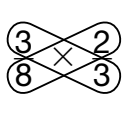
• Reducing Fractions, Part 2

- Reduce large numbers by **prime factorization**.

Cancel matching factors.

Example: $\frac{42}{105} = \frac{2 \cdot \cancel{3} \cdot 7}{3 \cdot 5 \cdot \cancel{7}}$ $\frac{2 \cdot \cancel{3} \cdot 7}{\cancel{3} \cdot 5 \cdot 7} = \frac{2}{5}$

- Cancel means “reduce before multiplying.”


 Pair any numerator with any denominator. $\frac{\cancel{3}}{8} \times \frac{2}{\cancel{3}} = \frac{1}{4}$ Cancel. (Reduce the pairs.)
 Multiply the reduced terms.

- Another way to find the greatest common factor (GCF) of two numbers:

- Write the prime factorization of each number.
- Circle the common factors.
- Multiply these common factors to find the GCF.

Example: Find the GCF of 24 and 64.

$$24 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 3$$

$$64 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot 2 \cdot 2$$

$$\text{GCF} = 2 \cdot 2 \cdot 2 = 8$$

Practice:

Reduce 1–3.

1. $\frac{72}{84}$ _____

2. $\frac{55}{150}$ _____

3. $\frac{39}{169}$ _____

Simplify 4 and 5.

4. $\frac{2}{5} \cdot \frac{7}{8} \cdot \frac{3}{14}$ _____

5. $\frac{5}{6} \cdot \frac{3}{4} \cdot \frac{12}{15}$ _____

Name _____

• **Dividing Fractions**

- The reciprocal of any whole number or fraction is the reverse of terms. The product of a term and its reciprocal is 1.

Examples: The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $\frac{1}{3}$ is 3, or $\frac{3}{1}$.

The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

- To divide whole numbers sometimes multiply by the **reciprocal** of the divisor.

Example: $24 \div 3 = \frac{24}{1} \div \frac{3}{1} = \frac{24}{1} \times \frac{1}{3} = 8$

- To divide fractions, multiply by the reciprocal of the divisor.

Example: $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = 2$

1. Copy the first number (fraction).
2. Change \div to \times .
3. Write the reciprocal of the second fraction.
4. Cancel (reduce pairs) if possible.
5. Multiply.

$$\left. \begin{array}{l} 1. \text{ Copy the first number (fraction).} \\ 2. \text{ Change } \div \text{ to } \times. \\ 3. \text{ Write the reciprocal of the second fraction.} \\ 4. \text{ Cancel (reduce pairs) if possible.} \\ 5. \text{ Multiply.} \end{array} \right\} \frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = 2$$

Practice:

Simplify 1–6.

1. $\frac{3}{16} \div \frac{1}{8}$ _____

2. $\frac{1}{5} \div \frac{7}{10}$ _____

3. $\frac{2}{3} \div \frac{3}{4}$ _____

4. $12 \div \frac{1}{6}$ _____

5. $\frac{8}{15} \div \frac{1}{3}$ _____

6. $\frac{6}{7} \div \frac{12}{14}$ _____

• Multiplying and Dividing Mixed Numbers

• To multiply and divide mixed numbers:

1. Change mixed numbers to improper (“top heavy”) fractions.
2. Then multiply or divide.
3. Simplify (reduce and/or convert) as necessary.

Examples:

Multiply

$$2\frac{1}{2} \times 1\frac{2}{3}$$

$$\downarrow \quad \downarrow$$

$$\frac{5}{2} \times \frac{5}{3}$$

Change mixed numbers to improper fractions.

$$\frac{5}{2} \times \frac{5}{3} = \frac{25}{6} \text{ Multiply.}$$

$$= 4\frac{1}{6} \text{ Simplify.}$$

Divide

$$3\frac{1}{3} \div 2\frac{1}{2}$$

$$\downarrow \quad \downarrow$$

$$\frac{10}{3} \div \frac{5}{2}$$

$$\downarrow \quad \downarrow$$

$$\frac{10}{3} \times \frac{2}{5} = \frac{4}{3}$$

Change mixed numbers to improper fractions.

Multiply by reciprocal of the divisor.

$$= 1\frac{1}{3} \text{ Simplify.}$$

Practice:

Simplify 1–6:

1. $4\frac{1}{2} \times \frac{1}{3}$ _____

2. $1\frac{1}{4} \cdot 7\frac{3}{5} \cdot 2\frac{1}{2}$ _____

3. $6 \times 4\frac{1}{4}$ _____

4. $1\frac{5}{8} \div 4$ _____

5. $8\frac{1}{6} \div 2\frac{1}{3}$ _____

6. $5 \div \frac{1}{5}$ _____

Name _____

Math Course 2, Lesson 27

- **Multiples**
- **Least Common Multiple**
- **Equivalent Division Problems**
- Use times table in Reference Guide to find the **least common multiple (LCM)** of small numbers.

Example: Find the LCM of 6 and 8.

Look down the 6's and the 8's columns.

Find the first number that is the same in both columns.

$$LCM = 24$$

- Use **prime factorization** to find the LCM of larger numbers.

Example: Find the LCM of 54 and 60.

1. Factor each number into prime factors.

$$54 = 2 \cdot 3 \cdot 3 \cdot 3$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

2. Write each prime factor the greatest number of times it was used to form either number.

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$

3. Multiply these factors to find the LCM.

$$LCM = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 = 540$$

- Use **equivalent division** to make complicated problems easier. Cancel matching zeros.
Multiply or divide the dividend and divisor by the same number to form a new problem that is easier to calculate.

Example: $\frac{220}{5}$

Multiply both numbers by 2 to form an equivalent problem.

$$\frac{220}{5} \times \frac{2}{2} = \frac{440}{10} = 44$$

Practice:

Find the least common multiple (LCM) of each pair or group of numbers.

1. 14 and 20 _____

2. 21 and 33 _____

3. 3, 7, and 10 _____

4. 2, 6, and 8 _____

5. 24 and 60 _____

6. 36 and 63 _____

- **Two-Step Word Problems**
- **Average, Part 1**

- **Two-step word problems** can be written as two-step computation problems.

Example: $10 - (6 + 3)$

Parentheses can make the problem easier.

Example: Julie went to the store with \$20. She bought 8 cans of dog food for 67¢ per can. How much money did she have left?

$$\$20 - (0.67 \times 8)$$

1. Find out how much she spent.

$$0.67 \times 8 = \$5.36$$

2. Then find out how much money she had left.

$$\$20 - \$5.36 = \$14.64$$

- Calculating an **average** is often a two-step process.
 1. Add the items.
 2. Divide by the number of items.

The answer must be *between* the smallest and the largest numbers.

Another name for average is **mean**.

Example: There were 3 people in the first row, 7 in the second row, and 20 in the third row. What was the average number of people in each of the rows?

$$\begin{array}{r}
 3 \text{ people} \\
 7 \text{ people} \\
 + 20 \text{ people} \\
 \hline
 30 \text{ people}
 \end{array}
 \quad
 \begin{array}{r}
 10 \text{ people per row} \\
 3 \text{ rows} \overline{)30 \text{ people}}
 \end{array}$$

- The average of two numbers is the number halfway between the given numbers.

Practice:

1. Hilda's scores on five games were the following:
83, 89, 94, 99, and 100. What was her average score? _____
2. Myrna bought 7 pounds of meat for a barbecue.
She paid \$2.49 per pound and gave the clerk a
\$20 bill. How much change should she receive? _____
3. Jim drove 175 miles in 3 hours 30 minutes.
How many miles per hour did Jim drive? _____
4. What is the average (mean) of 250 and 450? _____

Name _____

- **Rounding Whole Numbers**
- **Rounding Mixed Numbers**
- **Estimating Answers**

Before working a problem, **estimate** by *rounding* the numbers first.

- To round whole numbers:
 1. Circle the place value you are rounding to.
 2. Underline the digit to its right.
 3. Ask "Is the underlined number 5 or more?"
 - Yes → Add 1 to the circled number.
 - No → Circled number stays the same.
 4. Replace the underlined number (and any numbers after it) with zero.

Examples: $\textcircled{6}7 \rightarrow 70$

$\textcircled{3}28 \rightarrow 300$

- To round mixed numbers, compare each fraction to $\frac{1}{2}$.
If the fraction is equal to or **greater** than $\frac{1}{2}$, **round up** to the next whole number.
If the fraction is **less** than $\frac{1}{2}$, **round down**.

Examples: $6\frac{2}{3} \rightarrow 7$ (because $\frac{2}{3} > \frac{1}{2}$)

$6\frac{1}{8} \rightarrow 6$ (because $\frac{1}{8} < \frac{1}{2}$)

Practice:

1. Round 12,253 to the nearest thousand.

2. Round 27,390 to the nearest hundred.

3. Estimate the quotient when 73,280 is divided by 70.

4. Estimate the sum of $14\frac{2}{5}$ and $10\frac{1}{2}$.

- **Common Denominators**
- **Adding and Subtracting Fractions with Different Denominators**

- To add or subtract fractions with different denominators:

1. Find the least common multiple (LCM) of the denominators.

If the denominators are small numbers, use the times table.

Look down the columns of each denominator.

The first number that is the same in both columns is the LCM.

If the denominators are large numbers, use prime factorization.

Factor each denominator into prime factors.

Write each prime factor the greatest number of times it was used to form either number.

Multiply these factors to find the LCM.

2. **Rename** the fractions using the LCM as the new common denominator.

3. Add or subtract.

4. Simplify.

$$\begin{array}{r} \text{Example: } 3\frac{1}{2} = 3\frac{3}{6} \\ + 1\frac{1}{6} = 1\frac{1}{6} \\ \hline 4\frac{4}{6} = 4\frac{2}{3} \end{array}$$

Practice:

1. Rewrite $\frac{4}{7}$ and $\frac{3}{5}$ so they have

common denominators. _____

Then find the sum and simplify. _____

Simplify 2–4.

2. $1\frac{1}{10} + 5\frac{3}{4} + 3\frac{2}{5}$ _____

3. $3\frac{4}{5} - 1\frac{2}{10}$ _____

4. $\frac{3}{5} \cdot \frac{1}{6} + \frac{1}{4}$ _____

5. Compare: $\frac{1}{3} \bigcirc \frac{2}{5}$ _____