Name _____

- **Using Formulas**
- **Distributive Property**

- A **variable** is a letter that can stand for any number in a formula or other algebraic expression. Its value can vary.
- To evaluate expressions, replace the variables with numbers and simplify.
- **Distributive Property of Multiplication** applies over addition and subtraction.

Examples: $a(b + c) = ab + ac$ $\quad a(b - c) = ab - ac$

Distribute or "spread" the multiplication over the terms that are being added or subtracted inside the parentheses.

Example:

$$2(l + w) = 2l + 2w$$

Example: Simplify $5(7 - 3)$.

$$5(7 - 3) = 5 \cdot 7 - 5 \cdot 3 = 35 - 15 = 20$$

Example: Show two ways to simplify $6(20 + 5)$.

$6(20 + 5)$		$6(20 + 5)$
$6(25)$	or	$6 \cdot 20 + 6 \cdot 5$
150		$120 + 30$
		150

Practice:

Simplify 1–4.

1. $12(10 + 7)$ _____

2. $4(7 + a) - 3$ _____

3. $6(b - 4)$ _____

4. $10 + 3(1 + p)$ _____

Use each formula with the given measures.

5. $A = \frac{1}{2}bh$
Find A when $b = 12$ in. and $h = 4$ in. _____

6. $P = 2(l + w)$
Find P when $l = 0.5$ ft and $w = 3.1$ ft. _____

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• Repeating Decimals

- There are two possible results of decimal division.
  1. The division ends with a remainder of zero. It may be necessary to attach zeros to the dividend in order to continue dividing until it does end. (Be sure the dividend has a decimal point before attaching zeros.)

     Example: $\frac{3}{8} \longrightarrow 8\overline{)3.000}$ (quotient 0.375)

  2. The division does not end. Instead there is a pattern of repeating digits. Write a bar over the first pattern of repeating digits. **Repetend** is the name for repeating digits of a decimal number.

     Examples: Convert $\frac{43}{6}$ to a repetend $\longrightarrow 6\overline{)43.^{1}0^{4}0^{4}0^{4}0...} = 7.1\overline{6}$ (quotient 7.1666)

- We may choose to round a quotient. To round a repetend:
  1. Remove the bar and write the repeating digits.
  2. Then round as usual.

     Example: Round $25.\overline{405}$ to five decimal places.

     1. Remove the bar and write the repeating digits.

        $25.\overline{405} \longrightarrow 25.405405...$

     2. Then round to five places.

        2 5. 4 0 5 4 0 5 ... $\longrightarrow$ 25.40541

Practice:

1. Write 5.16333... with a bar over the repetend.

_____

2. Round $7.\overline{284}$ to the nearest thousandth.

_____

3. Divide 6.7 by 9 and write the quotient with a bar over the repetend.

_____

4. Divide 8.3 by 6 and write the quotient rounded to four decimal places.

_____

Name _____

- **Converting Decimals to Fractions**
- **Converting Fractions to Decimals**
- **Converting Percents to Decimals**

- To convert **decimals to fractions:**
  1. Write the digits after the decimal point as the numerator.
  2. Use the place value of the last digit to write the denominator.
  3. Reduce the fraction if possible.

  Example: $11.42 = 11\frac{42}{100} = 11\frac{21}{50}$

- To convert **fractions to decimals:**
  1. Divide the numerator by the denominator.
  2. Write the decimal point and zeros as necessary.
  3. Some fractions convert to repeating decimals.
  4. Show repeating decimals with a bar.

  Example: $\frac{2}{3} \longrightarrow 3\overline{)2.\ 0^{2}0^{2}0\ldots}$ (quotient $0.\ 6\ 6\ 6\ldots$) $\quad \frac{2}{3} = 0.\overline{6}$

- To convert **mixed numbers to decimals:**
  1. Whole number does not change.
  2. Divide the numerator by the denominator.

  Example: $3\frac{4}{5} = 3 + \frac{4}{5}$

  $5\overline{)4.0}$ (quotient 0.8) $\quad 3\frac{4}{5} = 3.8$

- To convert **percents to decimals:**
  Percents can be written as fractions or decimals.
  (Percent means "per hundred" or "hundredths.")

  Example: $5\% = \frac{5}{100} = 0.05$

Shortcut: Shift decimal point two places to the left.

$5\% \longrightarrow 5\% \longrightarrow 0.05$

Practice:

1. Write 43.75 as a reduced fraction. _____

2. Write $\frac{11}{6}$ as a decimal number. _____

3. Write 6.3% as a reduced fraction. _____

4. Write 148% as a decimal number. _____

5. Write $4\frac{1}{2}\%$ as a decimal number. _____

6. Write the product of $6\frac{2}{3} \times 1.25$ as a mixed number _____

• Division Answers

The answer to a division problem with a remainder can be written in three different ways:

- with a remainder $\quad 4\overline{)27}$ = 6 R 3
- as a mixed number (with the remainder as a fraction) $\quad 4\overline{)27}$ = $6\frac{3}{4}$ ← remainder, ← divisor
- as a decimal number $\quad 4\overline{)27.30}$ = 6.75 → 6.75

A division problem might require continued dividing until there is no remainder or rounding the answer as instructed.

Example: Round the quotient to the nearest thousandth.

$9\overline{)37.4050505050...}$ = 4.1555... → 4.156

Some division problems require rounding up or down to a whole number.

Example: Fifteen players will travel in vans. Each van carries six players.

a. How many vans are needed? three

b. How many vans will be full? two

Practice:

1. Divide 73 by 4 and write the answer

a. with a remainder. _____

b. as a mixed number. _____

c. as a decimal number. _____

2. Divide 19.6 by 6 and round the answer to three decimal places.

_____

3. One hundred forty-six musicians are seated in rows of fifteen.

a. How many rows of seats are needed? _____

b. If as many rows are filled as possible,
how many musicians will be in the row that is not full? _____

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• Dividing by a Decimal Number

- To divide by a decimal number:
  1. Move the decimal points.
     Make the **divisor** a whole number by moving the decimal point to the right. (The divisor is the number outside the division box.)

     Move the decimal point in the **dividend** the same number of places to the right. (The dividend is the number inside the division box.)

     Keywords to moving the decimal point are over, over, and up.

     Move the decimal point:
     over in the divisor (to make a whole number)
     over in the dividend (the same number of places as moved in the divisor)
     up to the quotient (straight up to the answer)
  2. Place a digit in the quotient above each digit of the dividend.
  3. Use zero as placeholder.
  4. Use short division with one-digit divisors.

Example: $0.06\overline{)3.36}$ ⟶ $\underbrace{006.}_{\text{over}}\overline{)\underbrace{336.}_{\text{over}}}$ up ⟶ $\begin{array}{r} 0\ 5\ 6. \\ 006.\overline{)3\ 3^{3}6.} \end{array}$

Practice:

Simplify 1–4.

1. 0.024 ÷ 0.6 _____

2. 1.4 ÷ 0.8 _____

3. 0.604 ÷ 0.2 _____

4. 1.2 ÷ 0.012 _____

5. Divide 4 by 0.18 and write the answer rounded to the nearest whole number. _____

Solve 6 and 7.

6. $0.6x = 35.7$ _____

7. $5.52 = 2.3t$ _____

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• Rates

- **Unit price** is the cost of one unit of a product.

Example: What is the unit price of a 24-ounce box of cereal that costs $3.60?

1. Multiply the numbers in the loop.
2. Divide by the outside number.

$$\frac{\$3.60}{24\text{ ounces}} = \frac{?}{1}$$

Unit price = $0.15 per ounce

- **Rate** is a ratio of two measurements.

Example: Hans pedaled 84 kilometers in 4 hours. What was his average speed?

Use this formula about distance, speed, and time:

distance = rate × time or **rate = $\frac{\textbf{distance}}{\textbf{time}}$**

Average speed $= \frac{84\text{ km}}{4\text{ hr}} = 21$ km/hr

- **Sales tax** is the tax charged on items purchased.

1. Multiply the purchase price by the tax rate.
2. Round to the nearest cent.

Example: What is the total cost of an $8.96 item plus 6% sales tax? Find the *amount of sales tax* by multiplying $8.96 by 0.06. To find the *total cost,* add the $0.54 tax to the $8.96 price.

$8.96	price	$8.96	price
× 0.06	tax rate	+ 0.54	tax (rounded)
$0.5376	tax	$9.50	total

Total cost = $9.50

Practice:

1. Claire's car averages 25 miles per gallon of gas.
At that rate, how far would it go on 10 gallons?

_____

2. A box of detergent cost $5.12 for 32 ounces. What is the unit price?

_____

3. The price of a radio was $14. The sales tax was $3\frac{1}{2}$%.
What was the total cost including sales tax?

_____

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• Powers of 10

- The exponent of a power of 10 tells the number of zeros in standard form.

  Example: $10^4 = 10{,}000$

- To *multiply* powers of 10, *add* the exponents.

  Example: $10^3 \times 10^4 = 10^{3+4} = 10^7$

- To *divide* powers of 10, *subtract* the exponents.

  Example: $10^6 \div 10^2 = 10^{6-2} = 10^4$

Rules of Exponents (for all powers with the same base)
$a^x \cdot a^y = a^{x+y}$
$\frac{a^x}{a^y} = a^{x-y}$
$(a^x)^y = a^{xy}$

Trillions			Billions			Millions			Thousands			Units (Ones)			
hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	Decimal point
10^{14}	10^{13}	10^{12}	10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	.

- Powers of 10 are used in expanded notation.

  Example: Write 5206 in expanded notation using powers of 10.

  $(5 \times 1000) + (2 \times 100) + (6 \times 1)$

  $(5 \times 10^3) + (2 \times 10^2) + (6 \times 10^0)$ (Remember that $10^0 = 1$.)

- To multiply by a positive power of 10:
  Shift the decimal point to the *right* the number of places shown by the exponent.

  Example: $3.14 \times 10^4 = 31{,}400$ (4 places)

- To divide by a positive power of 10:
  Shift the decimal point to the *left* the number of places shown by the exponent.

  Example: $3.5 \div 10^4 = 0.00035$ (4 places)

Practice:

Solve 1 and 2.

1. 0.238×10^3 _____

2. $1.536 \div 10^2$ _____

Write each missing exponent.

3. $(10^6)(10^2) = 10^{\square}$ _____

4. $(7^7) \div (7^{\square}) = 7^2$ _____

5. Write 1294 in expanded notation using powers of 10.

_____

Name _____

• Fraction-Decimal-Percent Equivalents

- To write a fraction or decimal as a percent, multiply by 100%.

  Examples: $\frac{7}{10} \times 100\% = \frac{70\cancel{0}}{1\cancel{0}} = 70\%$

  $0.8 \times 100\% = 80\%$

- To write a percent as a decimal or fraction, divide by 100%.

  Examples: $60\% \div 100\% = \frac{6\cancel{0}}{10\cancel{0}} = 0.6$

  $60\% \div 100\% = \frac{6\cancel{0}}{10\cancel{0}} = \frac{3}{5}$

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{3}{10}$	0.3	30%
$\frac{4}{10} = \frac{2}{5}$	0.4	40%

Practice:

1. Complete the table.

Fraction	Decimal	Percent
	0.2	
$\frac{2}{3}$		
		12%

2. Marie put three fifths of her stamps into an album.

What percent are not in the album? _____

3. Eduardo traded $\frac{1}{6}$ of his baseball cards for new ones.

Is that more or less than 0.25 of his cards? _____

Name _____

• Adding and Subtracting Mixed Measures

- A **mixed measure** uses two or more units of the same type of measurement.
- To add mixed measures:
  1. Line up matching units.
  2. Add like units.
  3. Simplify from right to left.

Example:

$$\begin{array}{rrrr} & 1\text{ yd} & 2\text{ ft} & 7\text{ in.} \\ + & 2\text{ yd} & 2\text{ ft} & 8\text{ in.} \\ \hline & 3\text{ yd} & 4\text{ ft} & 15\text{ in.} \end{array}$$

Simplify: Change 15 in. to 1 ft, 3 in. Add to the 4 ft. This now makes 3 yd, 5 ft, 3 in. Change 5 ft to 1 yd, 2 ft. Add to the 3 yd. This now makes 4 yd, 2 ft, 3 in.

- To subtract mixed measures:
  1. Line up matching units.
  2. Subtract from right to left.
  3. Borrow by converting from one measure to the next.

Example: Subtract 2 yd, 1 ft, 8 in. from 4 yd, 3 in.

$$\begin{array}{rrrr} & \overset{3}{\cancel{4}}\text{ yd} & & 3\text{ in.} \\ - & 2\text{ yd} & 1\text{ ft} & 8\text{ in.} \\ \hline \end{array} \longrightarrow \begin{array}{rrrr} & \overset{3}{\cancel{4}}\text{ yd} & \overset{2}{\cancel{(3}}\text{ ft)} & \overset{15}{\cancel{3}}\text{ in.} \\ - & 2\text{ yd} & 1\text{ ft} & 8\text{ in.} \\ \hline & 1\text{ yd} & 1\text{ ft} & 7\text{ in.} \end{array}$$

(3 ft)

Practice:

Add or subtract 1–4.

1.

$$\begin{array}{rrrr} & 6\text{ hr} & 29\text{ min} & 14\text{ sec} \\ + & 1\text{ hr} & 41\text{ min} & 12\text{ sec} \\ \hline \end{array}$$

2.

$$\begin{array}{rrrr} & 11\text{ yd} & & 9\text{ in.} \\ - & 10\text{ yd} & 2\text{ ft} & 10\text{ in.} \\ \hline \end{array}$$

3. 6 yd 3 ft 8 in. + 1 ft 9 in. _____

4. 2 lb 14 oz − 16 oz _____

Name _____

• Unit Multipliers and Unit Conversion

- **Unit multipliers** are ratios (or fractions) with units and are equal to 1.

Example: Write two unit multipliers for these equivalent measures.

3 ft = 1 yd

Write one measure as the numerator and its equivalent as the denominator.

$$\frac{3 \text{ ft}}{1 \text{ yd}} \text{ and } \frac{1 \text{ yd}}{3 \text{ ft}}$$

- Use unit multipliers to convert from one unit of measure to another.

Given measure	×	Unit multiplier	=	Converted measure

Units converting **to** a given measure are in the **numerator.**
Units converting **from** a given measure are in the **denominator.**

Example: Use a unit multiplier to convert 240 yards to feet.
Problem says change *to* ft. Use the unit multiplier that has ft in the *numerator.*

Cancel matching *units* and multiply.

$$240 \cancel{\text{yd}} \cdot \frac{3 \text{ ft}}{1 \cancel{\text{yd}}} = 720 \text{ ft}$$

Practice:

Use a unit multiplier for each conversion.

1. 7000 grams to kilograms _____

2. 15 feet to inches _____

3. 156 ounces to pounds (16 oz = 1 lb) _____

4. 75 meters to millimeters _____

5. 325 yards to feet _____

6. 112 centimeters to meters _____