• Scientific Notation for Large Numbers

Numbers used in science are often very large or very small. **Scientific notation** is a way to express numbers as a product of a decimal and a power of 10.

Example: $9{,}461{,}000{,}000{,}000 = 9.461 \times 10^{12}$

The power of 10 shows where the decimal point is located when the number is written in standard form.

- To write a large number in standard form:
  1. Shift the decimal point to the right the number of places shown by the positive exponent.
  2. Use zero as a placeholder.

Example: Write 4.26×10^6 in standard form.

$4.26 \times 10^6 \longrightarrow \underbrace{4260000}_{\text{6 places}} \longrightarrow 4{,}260{,}000$

- To write a large number in scientific notation:
  1. Place the decimal point to the right of the first non-zero digit.
  2. Use the power of 10 to show the real location of the decimal point.
  3. Omit terminal zeros.

Example: Write 405,700,000 in scientific notation.

$405{,}700{,}000 \longrightarrow 4.\underbrace{05700000}_{\text{8 places}} \longrightarrow 4.057 \times 10^8$

Practice:

Write each number in scientific notation.

1. 12 billion ________ **2.** 263,000 ________

Write each number in standard form.

3. 5.6×10^6 ________ **4.** 1×10^8 ________

Compare 5 and 6.

5. three hundred billion ◯ 3×10^6 **6.** 52,000,000 ◯ 5.2×10^7

Name ____________________

• Order of Operations

- When more than one operation occurs in the same expression, perform the operations in the order listed below.

Order of Operations
1. Parentheses, brackets, or braces
2. Exponents (powers) and roots
3. Multiply and divide, in order, left to right.
4. Add and subtract, in order, left to right.

- Another good way to remember the order of operations is with the sentence "**P**lease **e**xcuse **m**y **d**ear **A**unt **S**ally." Each initial letter stands for an order-of-operations word.

  Parentheses

  Exponents

  Multiplication and **D**ivision

- A division bar may serve as a symbol of inclusion. Simplify above and below the bar before dividing.

  Example: $\frac{3^2 + 3 \cdot 5}{2} = \frac{9 + 15}{2} = \frac{24}{2} = 12$

Practice:

Simplify 1–4.

1. $12 + 52 - 9 \div 3$ ________

2. $(100 - 7 \cdot 2 + 3) \div 10$ ________

3. $\frac{6 \div 2 + 3 \cdot 9}{\sqrt{100}}$ ________

4. $\frac{4^2 \cdot 4^3 + 3 \cdot 4}{4}$ ________

5. Evaluate $\frac{(a^2 + a) \cdot b}{\sqrt{c}}$ if $a = 5$, $b = 3$, and $c = 4$. ________

• Ratio Word Problems

- To solve ratio word problems:
  1. Make and complete a ratio box.
     Write given numbers in boxes.
     Write a letter in the box that answers the question asked.
  2. Write a proportion using the numbers in the ratio box.
  3. Solve the proportion.
     Cross-multiply.
     Divide by known factor.

Example: The ratio of salamanders to frogs was 5 to 7.

If there were 20 salamanders, how many frogs were there?

	Ratio	Actual Count
Salamanders	5	20
Frogs	7	*F*

$$\frac{\text{salamanders}}{\text{frogs}} = \frac{5}{7} = \frac{20}{F}$$

$$5 \cdot F = 7 \cdot 20$$

$$5F = 140$$

$$F = \frac{140}{5}$$

$$F = 28 \text{ frogs}$$

Practice:

1. The ratio of bats to balls was 8 to 20.
If there were 20 bats, how many balls were there?

____

2. The bread recipe calls for 8 cups of flour and 3 eggs.
The baker used 24 cups of flour to make bread.

How many eggs did she use? ___________________________

3. The ratio of violin players to cello players was 6 to 3.
The orchestra had 8 violin players.

How many musicians played the cello? ___________________________

Name ______________________________

• Rate Word Problems

- Rate is a ratio of two measurements.
- Rate can be stated in two ways—as a ratio and its reciprocal.
- Two ways to solve a rate problem:
  1. Multiply by the correct form of the rate.

     Choose the rate that allows you to cancel the units you want to change and to keep the units you want in your answer.
  2. Use the loop method.

Example: Eight ounces of the solution cost 40 cents.
Find the cost of 32 ounces of the solution.

1. Rate method:

   The two rates are $\frac{8 \text{ ounces}}{40 \text{ cents}}$ and $\frac{40 \text{ cents}}{8 \text{ ounces}}$.

   Use the rate that has cents in the numerator to change to cents.
   Cancel and multiply.

   $\overset{4}{\cancel{32}}\ \cancel{\text{ounces}} \times \frac{40 \text{ cents}}{\underset{1}{\cancel{8}}\ \cancel{\text{ounces}}}\ 160 \text{ cents} = \$1.60$

2. Loop method:

   $\frac{\text{ounces}}{\text{cents}} \quad \frac{8}{40} = \frac{32}{?} \longrightarrow (40 \times 32) \div 8 = 160 \text{ cents} = \$1.60$

Practice:

1. Eggs cost $5.07 for 3 dozen. What was the price per dozen?

____

2. If 30 pounds of bird seed cost $21, how much would 50 pounds cost at the same rate?

____

3. In Rosa's collection, the ratio of rings to bracelets is 2 to 3. The ratio of bracelets to necklaces is 2 to 6. If Rosa has 10 rings, how many necklaces does she have? (*Hint:* First find how many bracelets she has.)

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• Average and Rate Problems with Multiple Steps

- To find a **missing sum** of an average, use one of two formulas.
  1. average × number of items = missing sum
  2. $\text{number of items}\overset{\text{average}}{\overline{)\text{missing sum}}} \longrightarrow \begin{array}{r} \text{average} \\ \underline{\times \ \text{number of items}} \\ \text{missing sum} \end{array}$

- To find a **missing number** from an average, first find the sum. Then find the number needed to reach that sum.
  1. average × number of items = sum

     $\text{number of items}\overset{\text{average}}{\overline{)\text{missing sum}}} \longrightarrow \begin{array}{r} \text{average} \\ \underline{\times \ \text{number of items}} \\ \text{sum} \end{array}$
  2. sum − sum of known numbers = missing number

     $\begin{array}{r} \text{known number} \\ \text{known number} \\ \text{known number} \\ \underline{+ \ \text{known number}} \\ \text{sum of known numbers} \end{array} \longrightarrow \begin{array}{r} \text{sum} \\ \underline{- \ \text{sum of known numbers}} \\ \text{missing number} \end{array}$

- To find a **missing number** that makes a new average, find the sums and subtract.
  1. average *A* × number of items = sum *A*
  2. average *B* × total number of items = sum *B*
  3. sum *B* − sum *A* = missing number

  Example: After 4 games, Annette's average score was 89. What score does Annette need on her fifth game to bring her average up to 90?

  average *A* × number of items = sum *A* ⟶ 89 × 4 = 356
  average *B* × total number of items = sum *B* ⟶ 90 × 5 = 450
  sum *B* − sum *A* = missing number ⟶ 450 − 356 = 94

Practice:

1. The average of five numbers is 88. Four of the numbers are 100, 80, 75, and 89.

What is the fifth number? ________

2. Jose scored an average of 20 points in each of four games. Altogether, what

was the total number of points that he scored for all four games? ________

3. Mei's average score on the first six holes in a miniature golf game was 6.
Her average score on the next 12 holes was 3.
What was her average score on all 18 holes? ________

Name ________________________________

• Plotting Functions

- A **function** is a rule that tells the relationship between numbers. Every function has exactly one output number for every input number.

Example: Find the output when the input is multiplied by 2 or $y = 2x$.

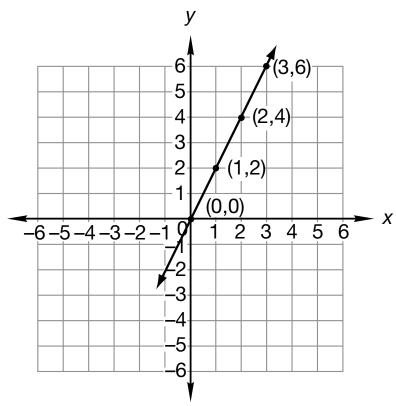
Input x	Output y
0	0
1	2
2	?
3	6

- The number pairs in a function table can be shown as coordinates of points on a coordinate plane. The graph of a function is a set of points.

When x is 2, y is 4.

Example: Graph the function $y = 2x$.

1. Create a table of (x, y) pairs using the rule.
2. Plot the pairs on the coordinate plane.
3. Draw a line through the points.

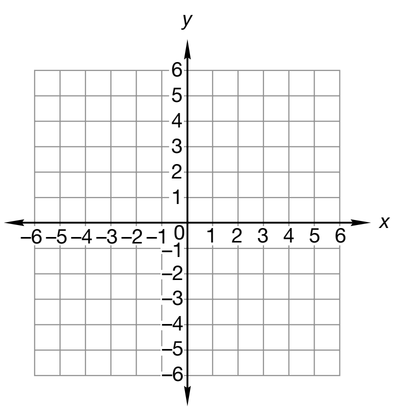


Practice:

Find the missing numbers in each table using the function rule. Then graph the (x, y) pairs on the coordinate plane and draw a line through the points.

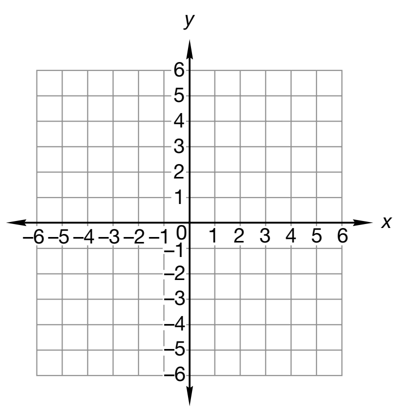
1. $y = 3x - 2$

x	y
0	
1	
2	
3	



2. $y = 2x + 1$

x	y
0	
1	
2	
3	



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• Negative Exponents
• Scientific Notation for Small Numbers

- Negative exponents can be used to express small numbers in scientific notation.

Examples: $10^{-1} = \frac{1}{10^1} = 0.1$

$10^{-2} = \frac{1}{10^2} = 0.01$

$10^{-3} = \frac{1}{10^3} = 0.001$

If $a \neq 0$, then
$a^0 = 1$
$a^{-n} = \frac{1}{a^n}$

- To write a small number in standard form:
  1. Shift the decimal point to the *left* the number of places shown by the negative exponent.
  2. Use zero as a placeholder.

Example: Write 6.32×10^{-7} in standard form.

.000000632 ⟶ 0.000000632

7 places to the left

- To write a small number in scientific notation:
  1. Place the decimal to the *right* of the first nonzero digit.
  2. Use the power of ten to show the real location of the decimal.

Example: Write 0.0000033 in scientific notation.

0000003.3 ⟶ 3.3×10^{-6}

6 places to the right

Example: Compare the following terms.

$0 \bigcirc 1 \times 10^{-3}$

$1 \times 10^{-3} = 0.001$

$0 \textcircled{>} 0.001$

$0 \textcircled{<} 1 \times 10^{-3}$

Practice:

Write each number in scientific notation.

1. 0.00009 ________________

2. 0.0000078 ________________

Write each number in standard form.

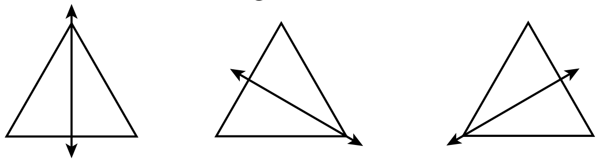
3. 2.6×10^{-4} ________________

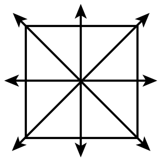
4. 3.21×10^{-3} ________________

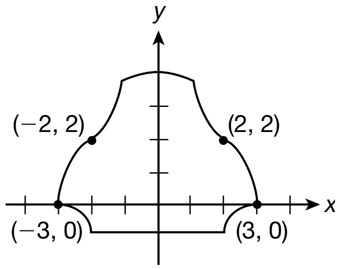
Name ____________________________________

• Symmetry

- A figure on paper has a **line of symmetry** if it can be divided in half so that the halves are *mirror images* of each other.







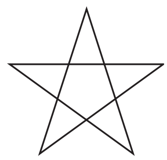
- The *y*-axis is the line of symmetry for the figure on the coordinate grid above. Corresponding points on each side of the line of symmetry are the same distance from the line but on opposite sides.
- A figure has rotational symmetry if it appears in its original position more than once in a full turn.

Example: Rotate this page to see these letters appear again after half a turn.

S H S

Practice:

1. Show all the lines of symmetry on this figure.



2. The points (0, 4) and (4, 0) are vertices of a square that has the *x*-axis and the *y*-axis as lines of symmetry. Draw the square on the graph provided. What points represent the other two vertices?

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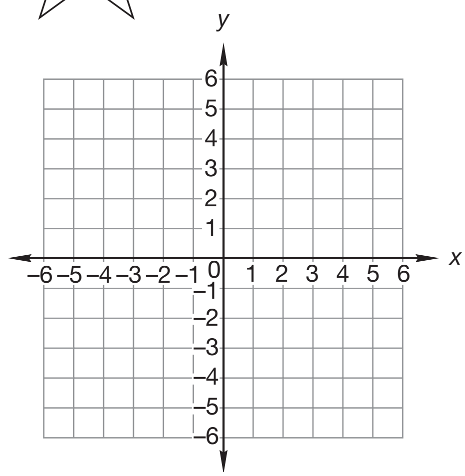
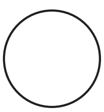
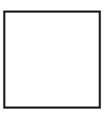
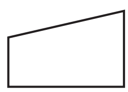


3. Which figure does not have rotational symmetry? __________

A.  **B.**  **C.**  **D.** 

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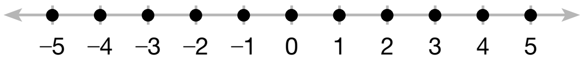
• Adding Integers on the Number Line

Integers: *all counting numbers, their opposites, and zero*
(does not include decimals or fractions)



The dots on this number line mark the integers from –5 through +5.

Signed numbers: *all numbers on a number line except zero*

Zero: *neither positive nor negative*
(The sum of two opposites is always zero.)

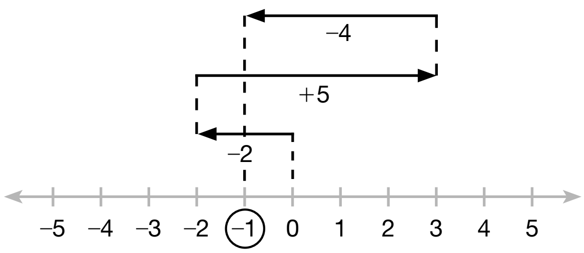
$$+3 + -3 = 0 \quad \text{or} \quad -3 + 3 = 0$$

Absolute value: *a number's distance from zero*
Writing a vertical bar on each side of a number shows absolute value.

$|3| = 3$ The absolute value of 3 equals 3.
$|-3| = 3$ The absolute value of –3 equals 3.

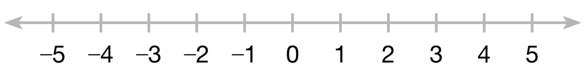
• To add integers on the number line:
1. Always begin the problem at zero.
2. Go right for positive.
3. Go left for negative.

Example: (–2) + (+5) + (–4)
1. Start at zero.
2. Count *left* 2 units. (–2)
3. Count *right* 5 units. (+5)
4. Count *left* 4 units. (–4)
The answer is –1.



Practice:

Use the number line to find each sum.



1. (–3) + (–3) ________

2. (–6) + (8) + (–1) ________

3. (+5) + (–7) + (–1) ________

Simplify 4 and 5.

4. $|3| \times |-1|$ ________

5. $|-4| \div |4|$ ________

Name ____________________________

• Fractional Part of a Number, Part 1
• Percent of a Number, Part 1

- To find a **fractional part of a number:**
  1. Translate the question into an equation.
     Replace "is" with $=$.
     Replace "of" with $\times$.
  2. Solve.

  Examples: What number is 0.6 of 31?

  $$W_N = 0.6 \times 31$$
  $$W_N = 18.6$$

  Three fifths of 120 is what number?

  $$\frac{3}{5} \times 120 = W_N$$
  $$72 = W_N$$

- To find a **percent of a number:**
  1. Change the percent to a decimal or fraction.
  2. Translate into an equation.
  3. Solve.

  Examples: Eight percent of \$3600 is commission.

  $$0.08 \times \$3600 = C$$
  $$\$288 = C$$

  What number is 25% of 88?

  $$W_N = \frac{1}{4} \times 88$$
  $$W_N = 22$$

Practice:

1. What is 40% of 50? ________

2. What is $66\frac{2}{3}\%$ of \$48? ________

3. What number is $\frac{7}{8}$ of 22? ________

4. Three fourths of 60 is what number? ________