Name ______________________________

• Using Proportions to Solve Percent Problems

Percent problems can be solved using proportions. If the problem is about parts of a whole, use three rows including a row for the total which is 100%.

- Make and complete a percent box.
  1. Write in the known facts.
  2. Write a letter in each unknown box.
- Write a proportion using **only** two of the rows.
  1. Use the row that answers the question asked.
  2. Use the row that is complete with two numbers.
- Solve the proportion.
  1. Cross-multiply.
  2. Divide by the known factor.

Example: Thirty percent of the class passed the test. If 21 students did not pass the test, how many students were in the class?

	Percent	Actual Count
Passed	30	*P*
Did not pass	70	21
Total	100	*T*

$$\frac{\text{did not pass}}{\text{total}} = \frac{70}{100} = \frac{21}{T}$$

$$70 \cdot T = 100 \cdot 21$$

$$70T = 2100$$

$$T = \frac{210\not{0}}{7\not{0}}$$

$$T = 30 \text{ students}$$

$$\begin{array}{r} 100\% \\ -\ 30\% \\ \hline 70\% \end{array}$$

$P = T - 21$
$P = 30 - 21$
$P = 9$ students

Practice:

1. Before it rained, only 25% of the plants in the desert garden were flowering. If 17 plants were not flowering before it rained, how many plants were in the garden in all? ________

2. Fifty-five percent of the 4000 customers placed orders, using the company's online service. How many customers did not order online? ________

3. The library bought 135 books in one month. If 20% of those books were fiction, how

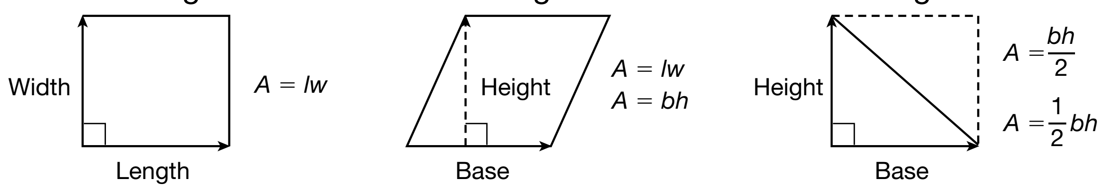
many books bought that month were nonfiction? ________

Name ______________________________

• Area of a Circle

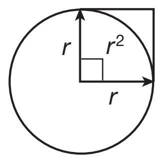
• To find the *areas* of some *polygons,* multiply the two perpendicular dimensions.

Rectangle — Parallelogram — Triangle



• To find the **area** of a circle, begin by multiplying two perpendicular dimensions.

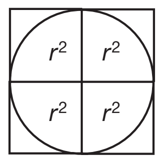
1. Multiply the radius by the radius.
   This equals the area of a square built on a radius.
   Area of square = r^2



   The area of the circle is **less** than the area of *four of these squares***,** but it is *more* than the area of *three squares.*

   The exact number that relates the area of a circle to its radius is π which is between 3 and 4. $\pi \approx 3.14$ or $\frac{22}{7}$.

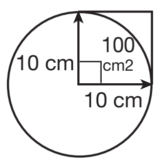
2. Multiply π times the area of the square built on the radius to find the area of the circle.



$A = \pi r^2$

Example: Find the area of a circle with a radius of 10 cm. Use 3.14 for π.
The area of a square built on the radius is 10 cm · 10 cm = 100 cm^2.
Multiply this by π.

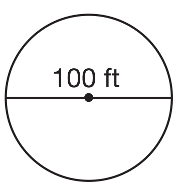
$A = \pi r^2$
$A \approx (3.14)(100 \text{ cm}^2)$
$A \approx 314 \text{ cm}^2$



Practice:

Find the area of each circle.

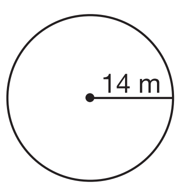
1.



Use 3.14 for π.

____

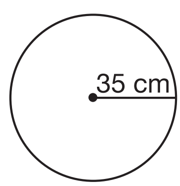
2.



Use $\frac{22}{7}$ for π.

____

3.



Use $\frac{22}{7}$ for π.

____

Name ______________________________

• Multiplying Numbers in Scientific Notation

- To multiply powers of 10, add the exponents.

  Example: $10^3 \cdot 10^4 = 10^{3+4} = 10^7$

- To multiply numbers in scientific notation:
  1. Multiply the decimal or whole numbers to find the decimal or whole number part of the product.
  2. Multiply the powers of 10 to find the power-of-10 part of the product.
  3. Write the expression in the proper form of scientific notation.

  Example: Multiply $(1.2 \times 10^5)(3 \times 10^7)$
  1. Multiply decimal (or whole) numbers.
     $1.2 \times 3 = 3.6$
  2. Multiply powers of 10.
     $10^5 \times 10^7 = 10^{5} + 7 = 10^{12}$
  3. The product is 3.6×10^{12}.

  Example: Multiply $(4 \times 10^6)(3 \times 10^5)$
  1. Multiply whole (or decimal) numbers.
     $4 \times 3 = 12$
  2. Multiply the powers of 10.
     $10^6 \times 10^5 = 10^{6+5} = 10^{11}$
  3. The product is 12×10^{11}.
     Rewrite the expression in the proper form of scientific notation.
     $(1.2 \times 10^1) \times 10^{11} = 1.2 \times 10^{12}$

Practice:

Multiply and write each product in scientific notation.

1. $(7.1 \times 10^3)(2.5 \times 10^2)$ ______________________

2. $(6.23 \times 10^{-5})(4 \times 10^{-3})$ ______________________

3. $(5 \times 10^{-3})(7 \times 10^4)$ ______________________

4. $(24.1 \times 10^6)(1.2 \times 10^{-6})$ ______________________

Name ___________________________

• Algebraic Terms

"Term" in arithmetic refers to the numerator or denominator of a fraction. It is customary to reduce fractions to lowest terms.

"Term" in algebra refers to a part of an algebraic expression or equation. An algebraic expression may have 1, 2, 3, or more terms.

Some Algebraic Expressions

Type of Expression	Number of Terms	Example
monomial	1	$-2x$
binomial	2	$a^2 - 4b^2$
trinomial	3	$3x^2 - x - 4$

- Each term has a signed number and may have one or more variables (letters). **When a term is written without a number, it is understood that the number is 1. When a term is written without a sign, it is understood that the sign is positive.**
- A term with no variable is called a **constant term.** Its value does not change. Constant terms can be combined by algebraic addition.
- Variable terms can also be combined by algebraic addition. They must be **like terms** (identical letter parts). Since –3xy and xy are like terms, –3xy + 1xy = –2xy.

Example: $3x + 2x^2 + 4 + x^2 - x - 1$ Collect like terms.

$\underbrace{2x^2 + x^2} + 3x - x + 4 - 1$ commutative property

$3x^2 + \underbrace{3x - x} + 4 - 1$ combined x^2 terms

$3x^2 + 2x + \underbrace{4 - 1}$ combined x terms

$3x^2 + 2x + 3$ combined constant terms

Notice that x and x^2 are **not** like terms and cannot be combined. Arrange terms in descending order of exponents. The term with the greatest exponent is on the left and the constant term is on the right.

Practice:

Simplify by combining like terms.

1. $4xy + xy - 7x + x$ ____________________

2. $6x - y - x - y$ ____________________

3. $14x^2 - y^2 + 3x^2 - y + 2y^2$ ____________________

Name ____________________

• Order of Operations with Positive and Negative Numbers

- To simplify expressions that have several operations, follow the order of operations.
  1. Simplify within parentheses, brackets, or braces.
  2. Solve for exponents.
  3. Multiply and divide, in order, left to right.
  4. Add and subtract, in order, left to right.

Example: Simplify $(-2) - [(-3) - (-4)(-5)]$.
There are only two terms (–2 and the quantity in brackets).
Use a slash to separate the two terms as they are simplified.

$(-2)\ /\ -[(-3) - (-4)(-5)]$
$(-2)\ /\ -[(-3) - (+20)]$
$(-2)\ /\ -(-23)$
$(-2)\ /\ +23$
$+21$

Practice:

Simplify 1–6.

1. $(-2) - (+8)(-4) - (-2)(-6)$ ____________________

2. $\dfrac{-3(-2) - 3(7)(-2)}{(-4)}$ ____________________

3. $\dfrac{(-12) - (-2)(-4)}{(-6) + (-6) - (+8)}$ ____________________

4. $\dfrac{(-2) - (5 \cdot 2) - (-4)^2}{(-2) + (-2)^2}$ ____________________

5. $(-2) - [2 - (-1)(-4)]$ ____________________

6. $(-6)^2 \div [(-4) \cdot (5 + 4)]$ ____________________

• Number Families

Rational Numbers

Counting numbers, their opposites, zero fractions, and decimals

Examples of rational numbers:

$-3, -\frac{1}{2}, -1, -\frac{3}{4}, -0.25, 0, \frac{1}{2}, 0.75, 2, \frac{9}{4}$

Integers

Counting numbers, their opposites, and zero

. . . , –4, –3, –2, –1, 0, 1, 2, 3, 4, . . .

Whole Numbers

Counting numbers and zero

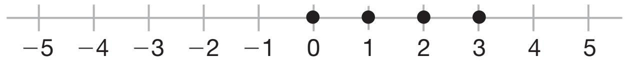
0, 1, 2, 3, 4, 5, . . .

Counting Numbers

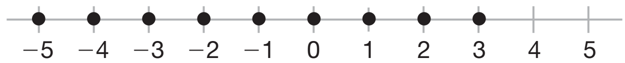
Numbers we use to count (no zero)

1, 2, 3, 4, 5, . . .

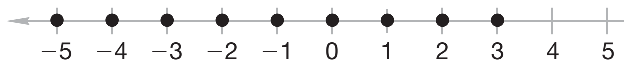
Example: Graph the integers that are less than 4. On a number line, draw a dot at every integer that is less than 4. Since the set of integers includes whole numbers, draw dots at 3, 2, 1, and 0.



The set of integers also includes the negatives of the positive whole numbers, so draw dots at –1, –2, –3, and so on.



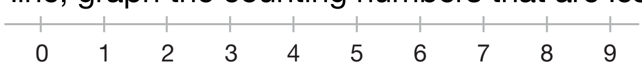
The arrowhead indicates that the graph of integers that are less than 4 continues without end.



Practice:

1. On the number line, graph the negative integers that are less than –2.

2. On the number line, graph the counting numbers that are less than 7.



3. True or false?

Every whole number is a counting number. ______________________

4. True or false?

Every fraction is a rational number. ______________________

Name ______________________________

• Multiplying Algebraic Terms

- To *add algebraic terms:*
  1. Add like terms.
  2. Adding the terms does not change the variable.

  Example: $3x + 2x = 5x$

- To **multiply algebraic terms:**
  1. List all factors.
  2. Multiply the numbers.
  3. Gather variable factors with exponents.
     All of the factors in the terms that are multiplied appear in the product.

  Example: $(3x)(2x) = 3 \cdot 2 \cdot x \cdot x$

  $= 6x^2$

  Terms may be multiplied even if they are not like terms.

  Example: $(-2x)(-3y) = 6xy$

  Example: Simplify $(-3x^2y)(2x)(-4xy)$.

$(-3) \cdot x \cdot x \cdot y \cdot (+2) \cdot x \cdot (-4) \cdot x \cdot y$	listed all factors
$(-3)(+2)(-4) \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$	commutative property
$(+24) \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$	multiplied numbers
$24x^4y^2$	gathered variable factors with exponents

Practice:

Simplify 1–6.

1. $(-3x)(-4xy)$ ______________________

2. $(6st)(-5s^2t)$ ______________________

3. $(-4ab)(b^2a)(7b)$ ______________________

4. $(2x)(-5y^2)(-3xy^2)$ ______________________

5. $(8r)(-2w^2)(r^2w^2)$ ______________________

6. $(11b^2)(-2c^2)(4b^2c^3)$ ______________________

• Multiple Unit Multipliers

Remember that a unit multiplier is a ratio equal to 1 composed of two equivalent measures.

Example: $\frac{3\text{ ft}}{1\text{ yd}}$ and $\frac{1\text{ yd}}{3\text{ ft}}$

More than one unit multiplier may be used without changing the measure since unit multipliers are equal to 1.

- To multiply by **multiple** (two or more) **unit multipliers:**
  1. Set up the problem with units changing **to** in the numerator.
  2. Cancel matching units of measurement.
  3. Multiply.

  Example: Use two unit multipliers to convert 5 hours to seconds.

  hours ⟶ minutes ⟶ seconds

  $$5\,\cancel{\text{hr}} \cdot \frac{60\,\cancel{\text{min}}}{1\,\cancel{\text{hr}}} \cdot \frac{60\text{ s}}{1\,\cancel{\text{min}}} = 18{,}000\text{ s}$$

- To convert **units of area:**

  Remember that units^2 = unit · unit.
  1. Set up the problem with units changing **to** in the numerator.
  2. Cancel matching units of measurement.
  3. Multiply.

  Example: Convert 1.2 m^2 to square centimeters.

  (m^2 = m · m)

  $$1.2\,\cancel{\text{m}^2} \cdot \frac{100\text{ cm}}{1\,\cancel{\text{m}}} \cdot \frac{100\text{ cm}}{1\,\cancel{\text{m}}} = 12{,}000\text{ cm}^2$$

- To convert **units of volume,** use three unit multipliers.

  Remember that units^3 = unit · unit · unit.
  1. Set up the problem with units changing **to** in the numerator.
  2. Cancel matching units of measurement.
  3. Multiply.

  Example: Convert 54 ft^3 to cubic yards.

  $$54\text{ ft}^3 \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} = 2\text{ yd}^3$$

Practice:

1. Use two unit multipliers to convert 36 square feet to square yards. __________

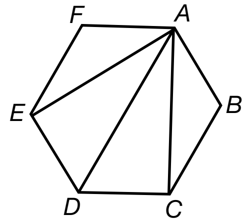
2. Use two unit multipliers to convert 3 m^2 to square centimeters. __________

3. Use three unit multipliers to convert 4 m^3 to cubic centimeters. __________

Name ________________________________

• Diagonals
• Interior Angles
• Exterior Angles

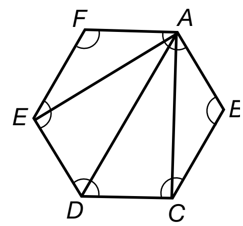
Diagonals of a polygon are segments that pass through the polygon and connect two vertices.



- Three diagonals can be drawn from one vertex to divide a regular hexagon into 4 triangles.

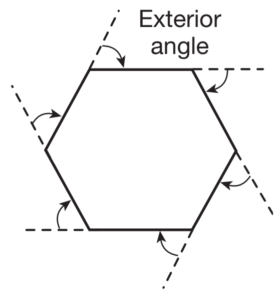
  The sum of the angles of one triangle measures 180°.
  The sum of the angles of four triangles measures 720°.

  Example: 180° + 180° + 180° + 180° = 720°



- The hexagon has six angles. The sum of all the angles is 720°. If all the angles are the same measure, then each angle of the hexagon is 120°, or 720° ÷ 6. These are all **interior angles.**

Exterior angles of a polygon are measured by following along the perimeter and measuring the amount of turn made at each corner.

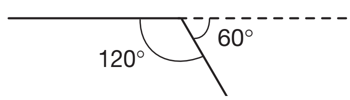


When following along the perimeter of a hexagon, the amount of turn made at the corner (instead of going straight) is the *exterior angle* of the hexagon at that vertex.

Going all the way around the hexagon completes one full turn of 360°.

- Six corners are turned when going around the hexagon.If all the angles are the same measure, then each exterior angle is 60°, or 360° ÷ 6.

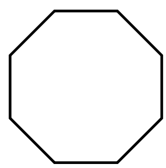
Notice that the interior and exterior angles are supplementary and total 180°.



If all of the turns are in the same direction, the sum of the exterior angles of *any* polygon is 360°.

Practice:

Answer problems 1–3 about a regular octagon.



1. How many diagonals can be drawn from one vertex?

Illustrate your answer. ________

2. How many triangles are formed by the diagonals

drawn from one vertex? ________

3. What is the measure of each interior angle and exterior angle? ________

- **Mixed-Number Coefficients**
  - **Negative Coefficients**

- To solve equations with **mixed-number coefficients:**
  1. First change the mixed number to an improper fraction.
  2. Multiply both sides of the equation by the **reciprocal** of the coefficient of x.

Example:

	$3\frac{1}{3}x = 5$	equation
	$\frac{10}{3}x = 5$	changed coefficient to improper fraction
	$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{10}}} \cdot \frac{\overset{1}{\cancel{10}}}{\underset{1}{\cancel{3}}}x = \frac{3}{\underset{2}{\cancel{10}}} \cdot \overset{1}{\cancel{5}}$	multiplied both sides by reciprocal of coefficient
	$x = \frac{3}{2}$	simplified

- To solve equations with **negative coefficients:**

Either divide both sides of the equation by the negative coefficient of x, or multiply both sides of the equation by the reciprocal of the negative coefficient of x.

Example: $-3x = 126$

Divide both sides by -3.		*Multiply* both sides by $-\frac{1}{3}$.
$-3x = 126$		$-3x = 126$
$\frac{-3x}{-3} = \frac{126}{3}$	or	$\left(-\frac{1}{3}\right)(-3x) = \left(-\frac{1}{3}\right)(126)$
$x = -42$		$x = -42$

Practice:

Solve 1–6.

1. $2\frac{1}{5}y = 66$ ____________

2. $-7x = 0.42$ ____________

3. $-4\frac{2}{3}a = 28$ ____________

4. $-12b = 60$ ____________

5. $6\frac{3}{4}w = 5.6$ ____________

6. $-\frac{7}{10}x = 9\frac{4}{5}$ ____________