• Evaluations with Positive and Negative Numbers

- When evaluating expressions with *negative* numbers, use parentheses to help prevent making mistakes with signs.

  Example: Evaluate $x - xy - y$ if $x = -2$ and $y = -3$.

  1. Write parentheses for each variable.
     $(\quad) - (\quad)(\quad) - (\quad)$
  2. Insert the numbers within the parentheses.
     $(-2) - (-2)(-3) - (-3)$
  3. Follow the order of operations.
     Multiply first.
     $(-2)(-3) = +6$
     Add algebraically from left to right.
     $(-2) - (+6) - (-3) = -5$

- Signed numbers are often written without parentheses.

  Example: Simplify $-3 + 4 - 5 - 2$.

  1. The easy way is to regroup the terms by their signs.
     $+4 - 3 - 5 - 2$
  2. Add the numbers with like signs.
     $-3 - 5 - 2 = 10$
  3. Add algebraically.
     $4 - 10 = -6$

- Another helpful way is to draw slashes between terms to make them easier to see.

  Example: Simplify $-2 + 3(-2) - 2(+4)$.

  1. Place the slash *before* each plus or minus sign that is not enclosed.
     $-2 \; / \; +3(-2) \; / \; -2(+4)$
  2. Add algebraically.
     $-2 - 6 - 8 = -16$

Practice:

1. If $x = -2$ and $y = 7x + 4$, then y equals what number? ________

2. Evaluate $a - ab + b$ if $a = -2$ and $b = 4$.

____

3. Evaluate $\frac{(a + b)^2}{(c - 1)}$ if $a = -1$, $b - 3$, and $c = 3$.

____

Simplify 4 and 5.

4. $-2 + 5 - 4 - 1$ ________

5. $5 - 6(-4) + 3(-1)$ ________

Name ______________________________

• Percent of Change

To find **percent of change:**

1. Use 100 as the *original* percent in a percent box.
2. Put other known facts into the "original," "change," and "new" boxes.
   If the change is an **increase, add** to the original.
   If the change is a **decrease, subtract** from the original.
3. Use the percent box to write a proportion.

Example: The county's population **increased** 15 percent from 1980 to 1990. If the population in 1980 was 120,000, what was the population in 1990?

	Percent	Actual Count
Original	100	120,000
+ Change	15	C
New	115	N

$$\frac{\text{original}}{\text{new}} = \frac{100}{115} = \frac{120{,}000}{N}$$
$$100 \cdot N = 115 \cdot 120{,}000$$
$$100N = 13{,}800{,}000$$
$$N = \frac{13{,}800{,}0\cancel{00}}{1\cancel{00}}$$
$$N = 138{,}000$$

Example: The price was **reduced** 30%. If the sale price was $24.50, what was the original price?

	Percent	Actual Count
Original	100	R
− Change	30	C
New	70	24.50

$$\frac{\text{original}}{\text{new}} = \frac{100}{70} = \frac{R}{24.50}$$
$$70 \cdot R = 24.50 \cdot 100$$
$$70R = 2450$$
$$R = \frac{245\cancel{0}}{7\cancel{0}}$$
$$R = \$35$$

Practice:

1. The cost of gasoline increased 75 percent in one year. If the cost after the increase was $2.45, what was the cost before the increase? __________

2. Gary increased his income by 30% after changing jobs. If his previous income was $25,000 per year, what was his income after the job change? __________

3. An electric frying pan was on sale for 20 percent off. The sale price was $36. What was the original price? __________

Name ___________________________________

• Two-Step Equations and Inequalities

- To solve some equations, isolate the variable using *two* steps.
  1. First add or subtract on both sides of the equation to isolate the variable term.
  2. Then multiply or divide on both sides to isolate the variable.

  Example: Solve $2x + 5 = 35$.

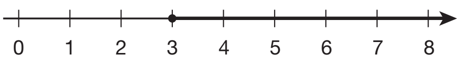
$2x + 5 - 5 = 35 - 5$	subtracted 5 from both sides
$2x = 30$	simplified
$\frac{2x}{2} = \frac{30}{2}$	divided both sides by 2
$x = 15$	simplified

- To solve inequalities, isolate the variable in the same way as two-step equations.

  Example: Solve $2x - 5 \geq 1$.

$2x - 5 + 5 \geq 1 + 5$	added 5
$2x \geq 6$	simplified
$\frac{2x}{2} \geq \frac{6}{2}$	divided by 2
$x \geq 3$	simplified

Graph the solution $x \geq 3$.



This graph indicates that all numbers greater than or equal to 3 satisfy the original inequality.

Practice:

Solve 1–4.

1. $5a - 35 = 60$ ____________________

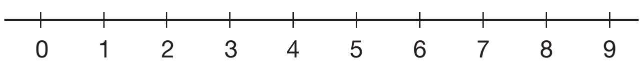
2. $2x + 24 = 30$ ____________________

3. $\frac{7}{8}t - 24 = 25$ ____________________

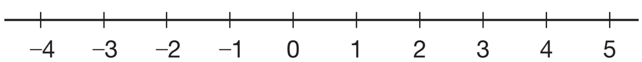
4. $0.5b + 4.7 = 23.7$ ____________________

Solve each inequality and graph the solution.

5. $4y - 6 \leq 10$ ____________________



6. $3x + 12 > 24$ ____________________



Name ____________________

• Probability of Dependent Events

- Two events are **dependent** if the outcome of one event depends on the outcome of the other.

Example: Joe has a bag of marbles. Some are red and some are black. Joe draws one marble and does not replace it.
The probability of drawing a red marble on the first draw is different than the probability of drawing a red marble on the second draw because the number and mix of marbles in the bag has changed.

- The probability of a series of dependent events occurring in a specific order is the **product** of the first event and the **recalculated probabilities** of each subsequent event.

Example: Six red marbles and five black marbles are in a bag. One marble is drawn and not replaced. A second marble is drawn. What is the probability that both marbles are red?

1. Calculate the probability of drawing a red marble.
   $P(\text{red}) = \frac{6}{11}$
   6 marbles are red out of a total of 11.
2. Assume the first marble drawn is red. Calculate the probability of drawing a second red marble.
   $P(\text{red}) = \frac{5}{10}$
   5 marbles are red out of a total of 10.
3. Find the product to calculate the probability of drawing 2 red marbles without replacement.
   $P(\text{red, red}) = \frac{6}{11} \cdot \frac{5}{10} = \frac{30}{110} = \frac{3}{11}$

Practice:

1. There are 4 red marbles and 2 green marbles in a bag.
Jill pulls out 1 marble and then another.
What is the probability that she pulls out 2 green marbles? ________

2. If 2 cards are drawn from a normal deck of 52 cards,
what is the probability that both cards will be spades? ________

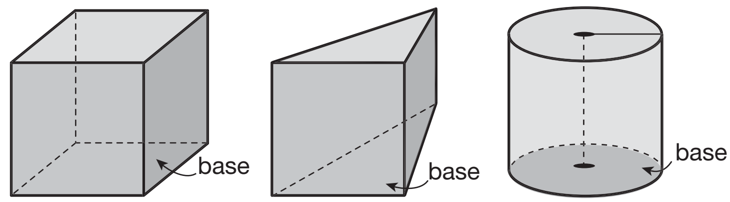
3. Three boxes on a table each contain a toy car. One car is red,
one is blue, and one is black. Sam chooses two boxes.
What is the probability that the first box he chooses
has a red car in it and the second box has a blue car in it? ________

• Volume of a Right Solid

A **right solid** is a geometric solid whose sides are *perpendicular* to the base.

If the base of a right solid is a polygon, the solid is called a prism.

If the base of a right solid is a circle, the solid is called a right circular cylinder.



Right square prism | Right triangular prism | Right circular cylinder

The **volume** of a right solid equals the area of the base times the height.

Volume = area of base × height ($V = Bh$)

Remember that volume is expressed in cubic units.

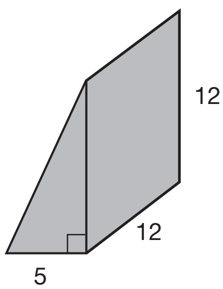
Example: Find the volume of a right circular cylinder with a base radius of 6 ft and height of 5 ft. Use 3.14 for π.

1. Find the area of the base.
   $A = (3.14)(62) = 113.04$ ft^2
2. Multiply the area of the base by the height.
   $V = 113.04 \times 5$
   $V = 565.2$ ft^3

Practice:

Find the volume of each figure. Dimensions are in centimeters.

1.



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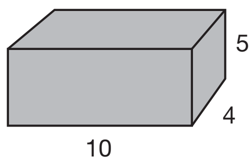
2.



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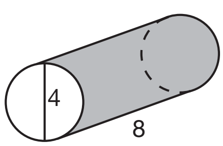
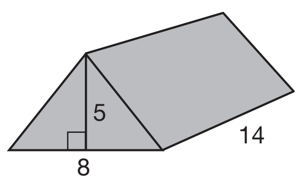
3.



Use 3.14 for π.

____

4.



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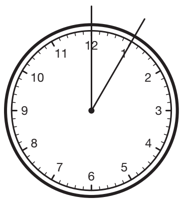
Name ____________________

- **Estimating Angle Measures**
- **Distributive Property with Algebraic Terms**

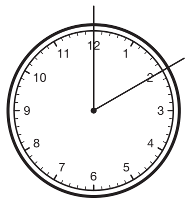
- The ability to measure an angle with a protractor is an important skill. The ability to **estimate** the measure of an angle is also a valuable skill.

  To estimate angle measure, use a mental image of a degree scale—a mental protractor. Use the face of a clock to "build" a mental image of a protractor.

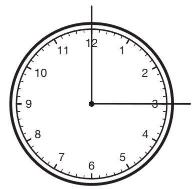
  A clock is a full circle, which measures 360°. There are 12 hours on the clock face. From one hour mark to the next is 30°.
  ($360° \div 12 = 30°$)



At 1 o'clock, the angle formed by the hands measures 30°.

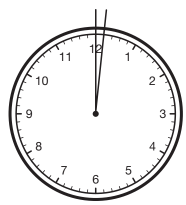


At 2 o'clock, the angle formed by the hands measures 60°.



At 3 o'clock, the angle formed by the hands measures 90°.

There are 60 minutes on the clock face. From one minute mark to the next is 6°.
($360° \div 60 = 6°$)



- The **distributive property** distributes multiplication over terms that are algebraically added.

  $a(b + c) = ab + ac$

  Examples: Simplify $2(x - 3) \longrightarrow 2 \cdot x - 2 \cdot 3 = 2x - 6$
  Simplify $-2(x + 3) \longrightarrow -2 \cdot x + (-2)(3) = -2x - 6$

Practice:

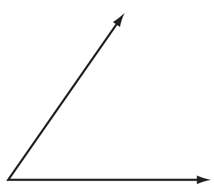
Simplify 1–4.

1. $5x + 2(x + 7)$ ________ **2.** $3y - 6(y - 2)$ ________

3. $-7x - 4(x - 3x)$ ________ **4.** $\frac{2}{3}y + 5(y^2 + y)$ ________

Estimate the measure of each angle. Then use a protractor to find the actual measure.

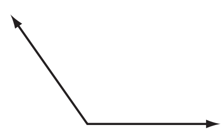
5.



estimate ________

actual measure ________

6.

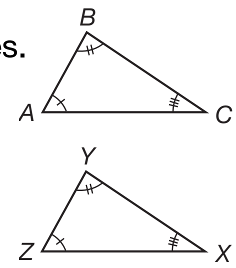


estimate ________

actual measure ________

- **Similar Triangles**
- **Indirect Measure**

Similar triangles have corresponding sides and corresponding angles.



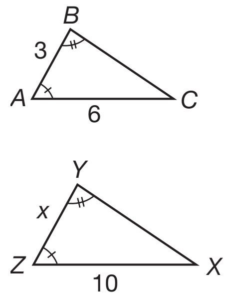
- Corresponding Sides
  - $\overline{AB}$ and $\overline{ZY}$
  - $\overline{BC}$ and $\overline{YZ}$
  - $\overline{CA}$ and $\overline{XZ}$
- Corresponding Angles
  - $\angle A$ and $\angle Z$
  - $\angle B$ and $\angle Y$
  - $\angle C$ and $\angle X$

> Corresponding angles of similar triangles have equal measures.
> The *lengths* of corresponding sides of similar triangles are *proportional*.

The *ratios* formed by corresponding sides of similar triangles are *equal*.

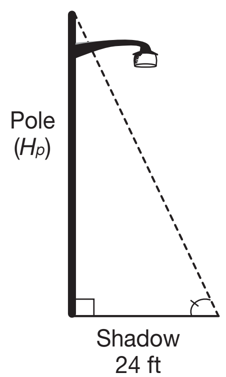
Example: Find the length of side *x*.

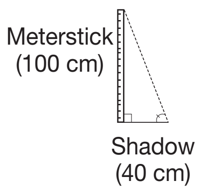
$\frac{6}{10} = \frac{3}{x}$	equal ratios
$6 \cdot x = 10 \cdot 3$	cross-multiplied
$\frac{6x}{6} = \frac{30}{6}$	divided by known factor
$x = 5$	solved



Use proportions to find the **indirect measure** of an unknown length or height.

Example: About how tall is the light pole?





	Meterstick	Pole
Height of object	100 cm	H_p
Length of shadow	40 cm	24 ft
	↑ cm	↑ ft

$$\frac{100}{40} = \frac{H_p}{24}$$

$$40 \cdot H_p = 24 \cdot 100$$

$$40H_p = 2400$$

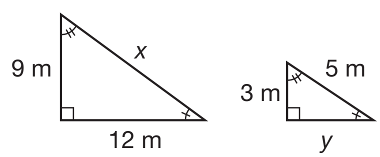
$$H_p = \frac{240\not{0}}{4\not{0}}$$

$$H_p = 60 \text{ ft}$$

Practice:

1. Find the length of side *x*. __________

2. Find the length of side *y*. __________



3. A man who is 6 ft tall has a 10-ft-long shadow. A nearby building casts a 100-ft-long shadow. How high is the building?

____

Name ______________________________

• Scale
• Scale Factor

- **Scale** is stated as a ratio. Scale models are examples of similar shapes.

Example: A model airplane is built on a scale of 1:24. If the wingspan of the model is 18 inches, the wingspan of the actual airplane is how many feet?

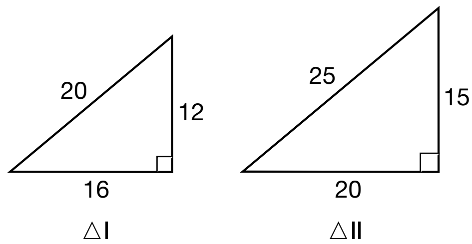
	Scale	Measure
Model	1	18
Object	24	*W*

$\longrightarrow \frac{1}{24} = \frac{18}{W} \longrightarrow W = 24 \cdot 18 \longrightarrow W = 432 \text{ in.}$

$\longrightarrow$ Then change to feet. $432 \cancel{\text{in.}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = 36 \text{ ft}$

- **Scale factor** is the number of times larger (or smaller) the dimensions of one figure are when compared to the dimensions of a similar figure. Express scale factor in decimal form.

Example: These two triangles are similar. Calculate the scale factor from the smaller triangle to the larger triangle.



To calculate the scale factor, divide corresponding sides.
Place side of figure going **to** in numerator. $\longrightarrow$
Place side of figure starting **from** in denominator. $\longrightarrow$ $\frac{25}{20} = 1.25$

The relationship between the *areas* of two similar figures is the scale factor *squared*.

Example: The area of △I is 1.252 times larger than the area of △II.

The relationship between the *volumes* of two similar figures is the scale factor *cubed*.

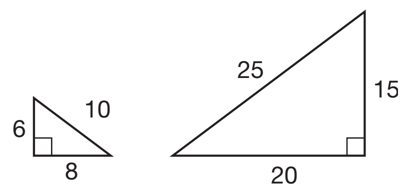
Example: The volume of △I is 1.253 times larger than the volume of △II.

Practice:

1. A model of a building is built on a scale of 1:400. The model stands 10 in. high.

   What is the height of the actual building? __________

2. What is the scale factor from the larger triangle to the smaller triangle? __________



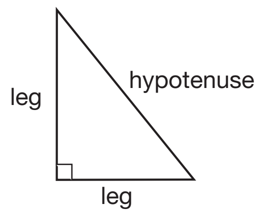
3. What is the scale factor from the smaller triangle to the larger triangle?

____

Name ____________________

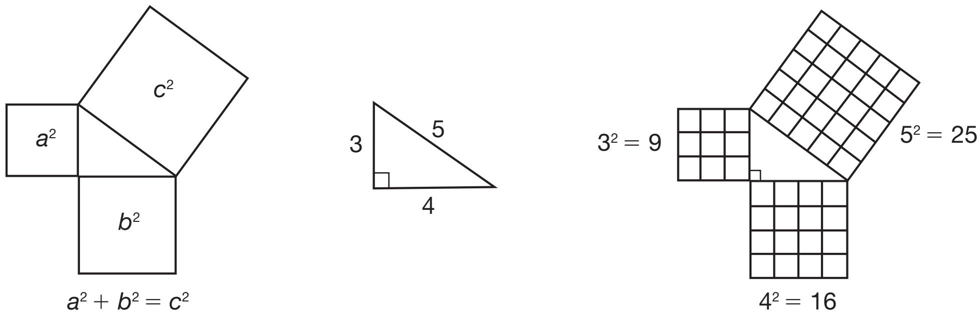
• Pythagorean Theorem

The *longest side* of a *right triangle* is called the **hypotenuse.**
The other two sides are called **legs.**



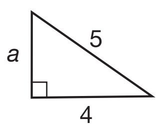
Every right triangle has a property that makes it special.
This property is called the **Pythagorean theorem,** which states:

The **area** of the square drawn on **the hypotenuse of a right triangle** equals the **sum** of the **areas** of the squares drawn on the **legs.**



The Pythagorean theorem is commonly expressed algebraically as $a^2 + b^2 = c^2$. Represent the two legs of a right triangle with a and b. Represent the hypotenuse with c. Use the Pythagorean theorem to find the missing length of a side of a right triangle.

Example: Use the Pythagorean theorem to find a.

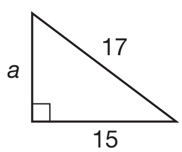


$$a^2 + b^2 = c^2$$
$$a^2 + 4^2 = 5^2$$
$$a^2 + 16 = 25$$
$$a^2 = 9$$
$$a = 3$$

Practice:

Use the Pythagorean theorem to find each missing length. Dimensions are in feet.

1.



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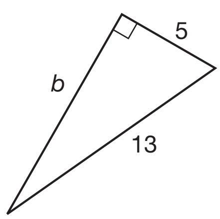

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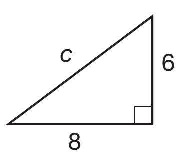


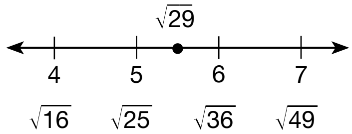
______________ ______________ ______________

Name ______________________________

• Estimating Square Roots
• Irrational Numbers

- Some counting numbers are perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, ...
- The square root of a number that is *between* two perfect squares can be estimated.

Example: Show the location of $\sqrt{29}$ on a number line.
Think of each number on the number line as the square root of a perfect square.



Since $\sqrt{29}$ is between $\sqrt{25}$ and $\sqrt{36}$, we graph $\sqrt{29}$ between 5 and 6.

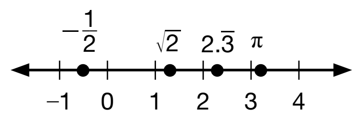
- **Irrational numbers** are numbers that cannot be expressed *exactly* as decimals or fractions.

Even when using the $\sqrt{2}$ key on a calculator, the answer will not be exact. Examples of irrational numbers include π, $\sqrt{2}$, and the square roots of counting numbers that are not perfect squares.

- Rational numbers are numbers that *can be* expressed exactly as decimals or fractions. (Repeating decimal numbers are rational.)
- The irrational numbers plus the rational numbers make up the set of **real numbers.**

Example: On a number line, show the approximate location of the points representing the following real numbers.
Then describe each number as rational or irrational.

$\pi \qquad \sqrt{2} \qquad 2.\overline{3} \qquad -\frac{1}{2}$



$\pi \approx 3.14$ Position π between 3 and 4, closer to 3. $\sqrt{2}$ is between $\sqrt{1}$ $(= 1)$ and $\sqrt{4}$ $(= 2)$. Position $\sqrt{2}$ between 1 and 2, closer to 1. Since $2.\overline{3} = 2\frac{1}{3}$, position $2.\overline{3}$ between 2 and 3, closer to 2. The rational numbers are $2.\overline{3}$ and $-\frac{1}{2}$. The irrational numbers are π and $\sqrt{2}$. (Neither π nor $\sqrt{2}$ can be expressed *exactly* as a fraction or a decimal.)

Practice:

1. Arrange in order from least to greatest: 4, $\sqrt{5}$, 3^2, −5 ______________

2. Which number in Exercise 1 is irrational? ________

3. Which of these numbers is between 6 and 8: $\sqrt{7}$, $\sqrt{60}$, or 2^2?