Name ______________________________

• Adding Integers

Integers include positive numbers, negative numbers, and zero. When we add two integers, the sign of the sum depends on the sign of both addends.

- The sum of **two positive** integers is always **positive.**
  Example: $(+4) + (+4) = +8$
- The sum of **two negative** integers is always **negative.**
  Example: $(-5) + (-3) = -8$
- The sum of two integers with different signs may be positive, negative, or zero.
- The sum of any integer and its opposite is always zero.
  Examples: $(-2) + (+2) = 0$ $(+5) + (-5) = 0$
- The sign of the sum of a positive integer and a negative integer that are not opposites depends on their absolute values. Remember: **Absolute value** is the distance from zero on the number line.
- To find the sum of two integers with different signs:
  1. Subtract the absolute values of the addends.
  2. Take the sign of the addend with the greater absolute value.

  Example 1: Find the sum. $(-2) + (+7)$
  First subtract the absolute values. $|-2| = 2$ $|+7| = 7$
  $7 - 2 = 5$
  Since the absolute value of $+7$ is greater than the absolute value of -2, the sign of the sum is positive. $(-2) + (+7) = +5$

  Example 2: Find the sum. $(+2) + (-7)$
  First subtract the absolute values. $|+2| = 2$ $|-7| = 7$
  $7 - 2 = 5$
  Since the absolute value of -7 is greater than the absolute value of $+2$, the sign of the sum is negative. $(+2) + (-7) = -5$

Practice:

Simplify.

1. $(-12) + (+4) =$ ________

2. $(+27) + (+3) =$ ________

3. $(-6) + (-7) =$ ________

4. $(-8) + (+8) =$ ________

5. $(-3) + (+9) =$ ________

6. $(+15) + (-19) =$ ________

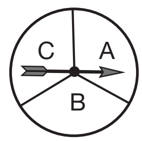
• Probability

- **Probability** is the likelihood that a particular event will occur. We can express probability as a fraction or decimal number between 0 and 1.
  - If an event is certain to occur, the probability is 1.
  - If an event is certain not to occur, the probability is 0.

  We can also express probability, or **chance**, as a percent between 0% and 100%.
- The probability that an event will not occur is the **complement** of the probability of the event occurring. The sum of the probability of an event *P*(E) and its complement *P*(not E) is always 1.
- **Theoretical probability** is based on analysis of a situation.
  To calculate the theoretical probability of an event, use this formula:

  $$P\text{ (Event)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

  Example: What is the probability that this spinner will stop in sector A or B?

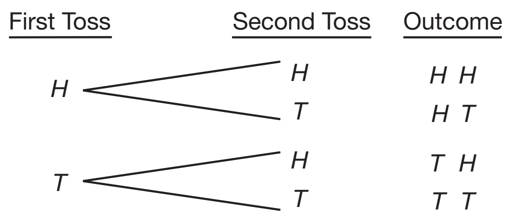
  There are 3 possible outcomes, but only 2 are favorable.

  $$P\text{(A or B)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{2}{3}$$

  You can also calculate *P*(A or B) as the sum *P*(A) + *P*(B).

  $$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

- The **sample space** of an experiment is the collection of all possible outcomes. This tree diagram shows the sample space for tossing a coin twice.

- **Experimental probability** is based on statistical data. To calculate the experimental probability of an event, use this formula:

  $$P\text{(Event)} = \frac{\text{number of favorable outcomes}}{\text{number of trials}}$$

- **Odds** are expressed as the ratio of favorable to unfavorable outcomes.

  Example: The odds that the spinner above will stop on A are 1:2.

Practice:

A number cube is rolled once. Find each probability or find the odds.

1. *P*(6) ____________________

2. *P*(not 7) ____________________

3. Odds of rolling an even number ____

4. *P*(2 or 3) ____________________

5. *P*(10) ____________________

6. Odds of rolling 4 ________________

• Subtracting Integers

Subtracting an integer is the same as adding its opposite.
The opposite of a positive integer is a negative integer with the same absolute value.
The opposite of a negative integer is a positive integer with the same absolute value.
The opposite of any integer is called the **additive inverse.**

Examples: The additive inverse of 4 is -4. The additive inverse of -6 is 6.

To subtract integers, replace the subtrahend with its opposite and then add.

Examples: $-3 - (-8)$ → $-3 + (+8) =$ **5** $-7 - (+3)$ → $-7 + (-3) =$ **−10**

Addition and Subtraction with Two Integers

To find the sum of addends with different signs: 1. Subtract the absolute values of the addends. 2. Take the sign of the addend with the greater absolute value.
To find the sum of addends with the same sign: 1. Add the absolute values of the addends. 2. Take the sign of the addends.
Instead of subtracting a number, add its opposite.

Practice:

Simplify.

1. $(-15) - (+4) =$ ____________

2. $(+7) - (+13) =$ ____________

3. $(-9) - (-5) =$ ____________

4. $(-8) - (+8) =$ ____________

5. $(-3) - (-4) =$ ____________

6. $(+10) - (-17) =$ ____________

Name ________________________________

• Proportions
• Ratio Word Problems

- A **proportion** tells us that two ratios are equal.

  Examples: $\frac{4}{20} = \frac{20}{100}$ $\frac{1}{2} = \frac{2}{4}$

- You can check to see if two ratios are equal by reducing each ratio to simplest form. Equal ratios reduce to the same fraction.

  Example: The proportion $\frac{6}{16} = \frac{12}{32}$ is true because $\frac{6}{16} = \frac{3}{8}$ and $\frac{12}{32} = \frac{3}{8}$.

- Another way to verify that a proportion is true is to find a constant factor for the numerator and the denominator that changes one ratio to the other.

  Example: $\frac{6}{16} = \frac{12}{32}$ because $\frac{6 \cdot 2}{16 \cdot 2} = \frac{12}{32}$.

- If you can identify a constant factor, you can find a missing number in a proportion.

  Examples:

  $\frac{5}{60} = \frac{x}{180}$

  $\frac{5 \cdot 3}{60 \cdot 3} = \frac{15}{180}$

  $\frac{35}{10} = \frac{7}{x}$

  $\frac{35 \div 5}{10 \div 5} = \frac{7}{2}$

- You can use a ratio box to help you solve ratio word problems.

  Example: The ratio of cats to dogs is 4 to 9. If there are 12 cats, how many dogs are there?

	Ratio	Actual
Cats	4	12
Dogs	9	x

$\frac{4}{9} = \frac{12}{x}$ $\left(\frac{4}{9}\right) \cdot \left(\frac{3}{3}\right) = \frac{12}{27}$

$x = 27$, so there are 27 dogs.

Practice:

Solve each proportion.

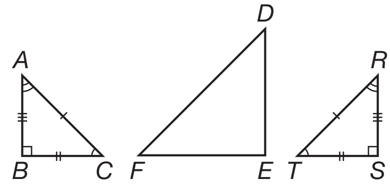
1. $\frac{3}{9} = \frac{15}{x}$ ____________ **2.** $\frac{x}{10} = \frac{16}{40}$ ____________ **3.** $\frac{6}{12} = \frac{3}{x}$ ____________

4. Two pounds of apples cost $3. How much do 6 pounds cost? ________

Name ____________________

• Similar and Congruent Polygons

Similar polygons have the same shape.
Congruent polygons have the same shape and size.
△*ABC* and △*DEF* are similar but not congruent.
△*ABC* ~ △*DEF*
△*ABC* and △*RST* are congruent. △*ABC* ≅ △*RST*

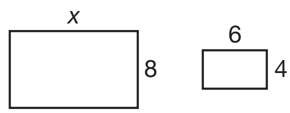


Name **corresponding parts** of similar or congruent polygons in the same order. The chart below names the corresponding parts and angles of △*ABC* and △*RST*.

Corresponding Angles	Corresponding Sides
∠*A* corresponds to ∠*R*	$\overline{AB}$ corresponds to $\overline{RS}$
∠*B* corresponds to ∠*S*	$\overline{AC}$ corresponds to $\overline{RT}$
∠*C* corresponds to ∠*T*	$\overline{BC}$ corresponds to $\overline{ST}$

	Similar Polygons	Congruent Polygons
corresponding angles	same measure	same measure
corresponding sides	proportional in length	same length

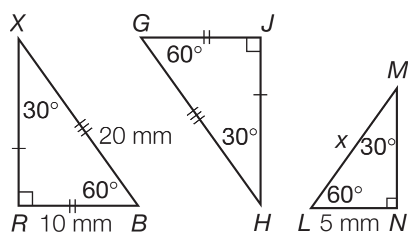
Use a proportion to find the missing side of a similar polygon.



Example: These two quadrilaterals are similar. What is the value of *x*?
Corresponding sides are proportional in length. So $\frac{6}{4} = \frac{x}{8}$.
Recall that to solve a proportion, you need to find a constant factor.
The constant factor for similar polygons is called the **scale factor.**
Here the scale factor is 2 because $4 \cdot 2 = 8$. So $x = 12$.

Practice:

Complete each statement.



1. △*BXR* ≅ ____________________

2. △*BXR* ~ ____________________

3. $\frac{BX}{XR} = \frac{GH}{}$ ____________________

4. The measure of $\overline{HG}$ = ____________________

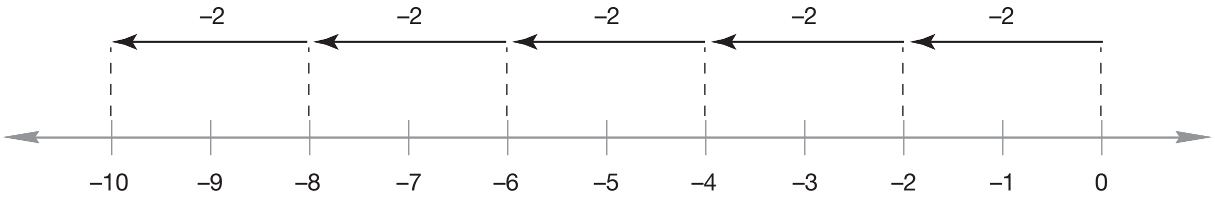
5. The scale factor from △*LMN* to △*BXR* is ____________________

6. The measure of $\overline{LM}$ = ____________________

Name ______________________________

• Multiplying and Dividing Integers

We can use the number line to multiply and divide integers.



Notice that $5(-2) = -10$.
So $-10 \div 5 = -2$ and $-10 \div (-2) = 5$.

Recall that -5 and 5 are opposites.
Since $5(-2) = -10$, the product $-5(-2)$ equals the opposite of -10, or 10.

Follow these rules for multiplying and dividing two integers:

1. Multiply or divide as indicated.
2. If the two numbers have the **same sign,** the answer is positive.
   If the two numbers have **different signs,** the answer is negative.

   Examples: $(-3)(-2) = 6$ $(-4)(3) = -12$

   Since a pair of negative factors is positive, the product of an even number of negative factors is positive. The product of an odd number of negative factors is negative.

   Example: Simplify $(-2)^5$.

   Write the exponent as the product of factors. $(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)$
   The number of negative factors is odd. $= 4 \cdot 4 \cdot (-2)$
   The product is negative. $= 16 \cdot (-2)$
   $= -32$

Practice:

Simplify.

1. $(-30)(10)$ ______________

2. $(-11)(-5)$ ______________

3. $63 \div (-9)$ ______________

4. $-144 \div -12$ ______________

5. $(-8)^2$ ______________

6. $(-1)^6$ ______________

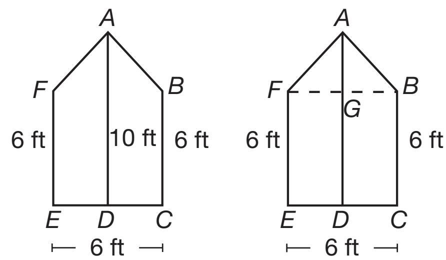
• Areas of Combined Polygons

You can use what you know about finding the areas of triangles and rectangles to find the areas of other polygons.
Recall the formulas below:

Area of a Triangle: $A = \frac{1}{2}bh$, where b = the base and h = the height.

Area of a Rectangle: $A = lw$, where l = length and w = width.

Example: Find the area of figure *ABCDEF.*

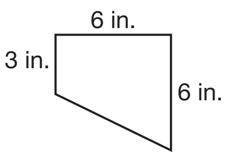


1. Draw $\overline{FB}$ to form the square *FBCE.* Label point *G,* the intersection of $\overline{FB}$ and $\overline{AD}$.
2. Calculate the area of *FBCE.* $A = s^2 = (6 \text{ ft})^2 = 36 \text{ ft}^2$
3. Find *AG.* Since $AD = 10$ ft and $GD = 6$ ft, $AG = 4$ ft
4. Calculate the area of $\triangle FAB$. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \text{ ft} \cdot 4 \text{ ft} = 12 \text{ ft}^2$
5. Find the total area of *ABCDEF*.
   Area of *ABCDEF* = Area of square *FBCE* + Area of $\triangle FAB$
   $= 36 \text{ ft}^2 + 12 \text{ ft}^2 = 48 \text{ ft}^2$

Practice:

Find the area of each figure.

1.



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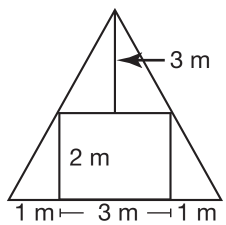
2.



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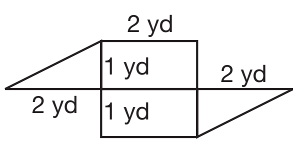
3.



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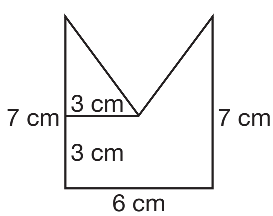
4.



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• Using Properties of Equality to Solve Equations

When you solve an equation, you must always perform the same operation on both sides of the equation to keep it balanced.

Inverse operations can help you solve an equation by **isolating the variable** on one side of the equation.

Addition and subtraction are inverse operations. $n + 3 - 3 = n$

Multiplication and division are inverse operations. $n \div 3 \cdot 3 = n$

Example:

Solve the equation. $y - 1.8 = 3.4$

Recall that inverse operations "undo" each other.

So to undo subtracting 1.8 from *y*, we add 1.8 to *y*. $y - 1.8 + 1.8 = 3.4 + 1.8$

To keep the equation balanced, we also add 1.8 to 3.4.

Simplify both sides to solve for *y*. $y = 5.2$

Example:

Solve the equation. $3 = \frac{3}{4}y$

To undo multiplying *y* by $\frac{3}{4}$, we divide *y* by $\frac{3}{4}$.

Recall that dividing by $\frac{3}{4}$ is the same as multiplying by $\frac{4}{3}$. To keep the equation balanced, we also multiply 3 by $\frac{4}{3}$. $3 \cdot \frac{4}{3} = \frac{3}{4}y \cdot \frac{4}{3}$

$4 = y$

Simplify both sides to solve for *y*.

Reverse the equation, using the Symmetric Property. $y = 4$

Practice:

Solve each equation.

1. $x + \frac{1}{3} = \frac{7}{9}$ ____________

2. $8m = 6.4$ ____________

3. $35 = \frac{r}{0.7}$ ____________

4. $x - 12 = 10.5$ ____________

5. $\frac{7}{8} = \frac{1}{2}x$ ____________

6. $8t = 60$ ____________

7. $w - \frac{1}{3} = \frac{1}{2}$ ____________

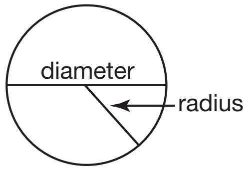
8. $\frac{2}{3}x = 6$ ____________

9. $1.2 + y = 4$ ____________

Name ____________________

• Circumference of a Circle

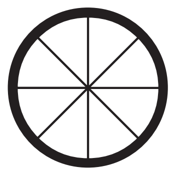
The **circumference** of a circle is a measure of the distance around the circle. So the circumference of a circle is like the perimeter of a polygon.



The circumference of a circle is about 3 times as long as its diameter. This relationship is represented by the number π.
Because π cannot be expressed exactly as a decimal or fraction, it is called an **irrational number**. We can estimate π as either 3.14 or $\frac{22}{7}$.
Recall that the diameter of a circle is twice the radius. So there are two formulas that we can use to calculate the circumference of a circle:

$$C = \pi d \text{ or } C = 2\pi r.$$

Example: A wheel has a diameter of 35 inches. About how far will the wheel travel in one complete turn?



The circumference is the distance that a wheel travels in a single turn.
Since we know the diameter, use the formula $C = \pi d$.

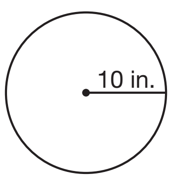
Notice that the number 35 is a multiple of 7. So it is easier to use the fractional representation for π.
The wheel will travel about 110 inches in one turn.

$$C = \pi \cdot 35$$
$$C \approx \frac{22}{7} \cdot 35$$
$$\approx 22 \cdot 5$$
$$\approx 110 \text{ in.}$$

Practice:

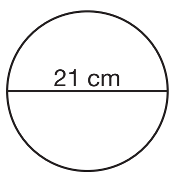
Find each circumference.

1.



Use 3.14 for π ____________________

2.



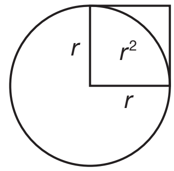
Use $\frac{22}{7}$ for π ____________________

3. The clock on Jill's wall has a radius of 6 inches. About how long a piece of ribbon does Jill need to put ribbon around the edge of the face of the clock? Round to the nearest inch.

____

4. A bicycle tire with a diameter of 28 inches rolls about how many inches in one turn? Use $\frac{22}{7}$ for π. ____________________

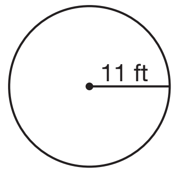
• Area of a Circle



We compare the area of a circle to the area of a square on its radius. The area of the circle is π times the area of the square.

$$A = \pi r^2$$

Example: Find the area of this circle in terms of π. Then find the area to the nearest square foot.



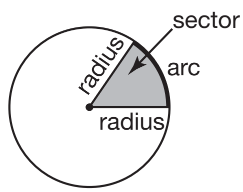
The radius of the circle is 11 feet.
Simplify the expression.
The area of the circle is 121π square feet.

$$\begin{aligned} A &= \pi \cdot (11 \text{ ft})^2 \\ &= \pi \cdot 121 \text{ ft}^2 \\ &= 121\pi \text{ ft}^2 \end{aligned}$$

Recall that the approximate value of π is 3.14 or $\frac{22}{7}$.
Use either value for π to find the area of the circle to the nearest square foot.
The area of the circle is about 380 square feet.

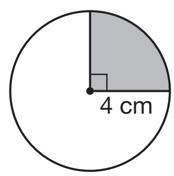
$$\begin{aligned} A &\approx 3.14 \cdot 121 \text{ ft}^2 \\ &\approx 379.94 \text{ ft}^2 \\ &\approx 380 \text{ ft}^2 \end{aligned}$$

A **sector** of a circle is a part of the interior of the circle. A sector is enclosed by two radii and an **arc** on the circle.



To find the area of a sector, first use the measure of the **central angle** to find what fraction of the whole circle the sector occupies.

Example: Find the area of the sector in terms of π.
The sector forms a central angle of 90°.
Recall that a full circle has 360° around the center.
$\frac{90°}{360°} = \frac{1}{4}$ So the area of the sector is $\frac{1}{4}\pi r^2$.



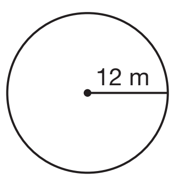
$$\begin{aligned} A &= \pi r^2 \\ &= \pi \cdot 4^2 \\ &= 16\pi \end{aligned}$$

The radius of the circle is 4 cm.
The area of the circle is 16π cm^2.
The area of the sector is $\frac{1}{4} \cdot 16\pi$ cm^2, or 4π cm^2.

Practice:

Find the area of each circle or shaded sector in terms of π.

1.



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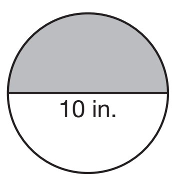

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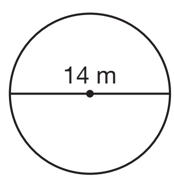

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