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Math Course 3, Lesson 41

• Functions

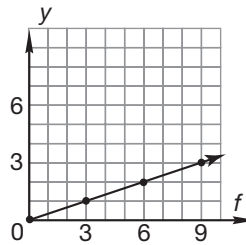
A **function** is a rule that identifies a relationship between a set of input numbers and a set of output numbers. A function rule can be described in words, expressed as an equation, or illustrated in a table or graph.

A function rule uses an **input** number to generate an **output** number. Each input number has exactly one output number.

Example: There are 3 feet in every yard. So dividing the number of feet you have by 3 tells you how many yards you have. The equation $y = \frac{1}{3}f$ expresses this relationship. The table below lists some pairs of numbers that relate yards to feet. The graph below shows the pairs of numbers on a coordinate plane. The x -axis represents feet (f) and the y -axis represents yards (y).

$$y = \frac{1}{3}f$$

f	y
0	0
3	1
6	2
9	3



If all the input-output pairs in a function align on a graph, the function is called a **linear function**.

Notice that the graph of the function $y = \frac{1}{3}f$ includes the origin $(0, 0)$. It also rises to the right on the coordinate plane. When a linear function has these characteristics, it is called a **proportional relationship**.

Every pair of numbers in a proportional relationship is related by the same constant. In the function $y = \frac{1}{3}f$, that constant value is $\frac{1}{3}$.

Not all functions are linear. For example, some functions have pairs of numbers that form a curve when they are graphed on the coordinate plane.

Practice:

Write an equation for each relationship. Then make a table of input-output pairs and tell whether the function is proportional.

1. A box of cereal costs \$2. _____
2. The second number is 4 more than the first number. _____
3. The circumference of a circle is the diameter times π . _____
4. There are 4 quarts in every gallon. _____

• Volume

The **volume** of a solid is the total amount of space it occupies or encloses. Volume is measured in cubic units, such as cubic feet (ft^3).

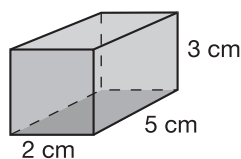
The **volume of any prism** equals the area of the base times the height.

Use the formula $V = Bh$.

The area of the base of a **rectangular prism** equals length (l) times width (w). So the formula for the volume of a rectangular prism is $V = lwh$.

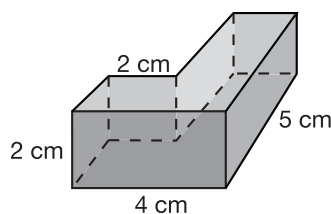
A **cube** is a special kind of rectangular prism with sides of equal length.

The volume of a cube is $V = s^3$ where s is the length of a side.

Examples:

This is a rectangular prism.

$$\begin{aligned} V &= lwh \\ &= 2 \text{ cm} \cdot 5 \text{ cm} \cdot 3 \text{ cm} \\ &= 30 \text{ cm}^3 \end{aligned}$$



This figure is made of two prisms. First use the sketch of the base to find the area of the base of each prism.

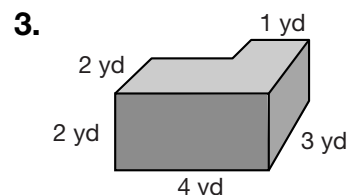
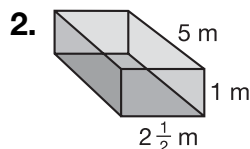
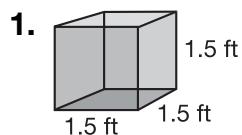
$$\begin{aligned} \text{Area of square base} &= s^2 = (2 \text{ cm})^2 = 4 \text{ cm}^2 \\ \text{Area of rectangular base} &= lw = 2 \text{ cm} \cdot 5 \text{ cm} \\ &= 20 \text{ cm}^2 \\ \text{Area of total base} &= 4 \text{ cm}^2 + 20 \text{ cm}^2 = 24 \text{ cm}^2 \end{aligned}$$

The volume equals the area of the base times the height.

$$V = Bh = 24 \text{ cm}^2 \cdot 2 \text{ cm} = 48 \text{ cm}^3$$

Practice:

Find the volume of each solid.



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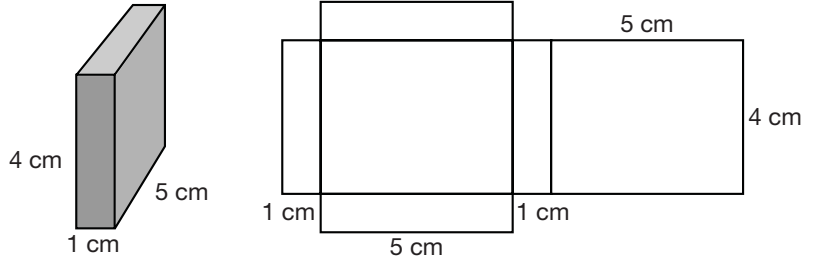
• **Surface Area**

The **surface area** of a solid is the total area of all its faces or surfaces.

The **net** of a solid is a two-dimensional drawing showing all the surfaces.

Example: Find the surface area of this rectangular prism.

The surface area of the prism is the total area of all the faces. Use the net to calculate the area of each face.



$$\text{Area} = \text{length} \times \text{width}$$

Area of front: $4 \text{ cm} \times 1 \text{ cm} = 4 \text{ cm}^2$

Area of back: $4 \text{ cm} \times 1 \text{ cm} = 4 \text{ cm}^2$

Area of left side: $4 \text{ cm} \times 5 \text{ cm} = 20 \text{ cm}^2$

Area of right side: $4 \text{ cm} \times 5 \text{ cm} = 20 \text{ cm}^2$

Area of top: $5 \text{ cm} \times 1 \text{ cm} = 5 \text{ cm}^2$

Area of bottom: $5 \text{ cm} \times 1 \text{ cm} = 5 \text{ cm}^2$

Total surface area: 58 cm^2

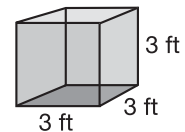
The **lateral surface area** of a solid does *not* include the bases. One way to find the lateral surface area of a prism is to find the perimeter of the base and multiply by the height.

Example: Find the lateral surface area of this cube.

The base of a cube is a square.

The perimeter of the base = $3 \text{ ft} \times 4 = 12 \text{ ft}$

Lateral surface area = perimeter \times height
 $= 12 \text{ ft} \times 3 \text{ ft} = 36 \text{ ft}^2$



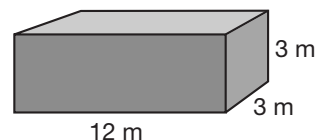
Practice:

1. Find the surface area of a cube with edges 5 inches long. _____

2. What is the total surface area of this prism? _____

3. What is the lateral surface area of the prism? _____

4. The perimeter of a garage is 80 ft. The height is 10 ft.



What is the lateral surface area of the garage? _____

• Solving Proportions Using Cross Products

A **proportion** is a statement that two ratios are equal. **Cross products** in a proportion are always equal.

Example: $\frac{3}{4} = \frac{15}{20}$

Equal ratios have equal cross products. $\frac{3}{4} = \frac{15}{20}$

So $3 \times 20 = 4 \times 15$.

$60 = 60$

You can use cross products to find the unknown term in a proportion.

Example: Solve for x . $\frac{x}{16} = \frac{5}{8}$

$8x = 16 \times 5$

Write the cross products.

$\frac{8x}{8} = \frac{(16 \times 5)}{8}$

Divide both sides by 8 to isolate the variable.

$x = 10$

Simplify to find the value of x .

Example: If 3 pounds of hamburger cost \$7.50, how much will 5 pounds cost?

First write the proportion, using a ratio table. Then use cross products to find x .

	Case 1	Case 2
Number of Pounds	3	5
Cost	\$7.50	x

$\frac{3}{\$7.50} = \frac{5}{x}$

$3x = 5 \times \$7.50$

$\frac{3x}{3} = \frac{(5 \times \$7.50)}{3}$

$x = \$12.50$ So 5 pounds of hamburger will cost \$12.50.

Practice:

Use cross products to solve.

1. $\frac{y}{3.5} = \frac{9}{10.5}$

2. $\frac{11}{13} = \frac{x}{52}$

3. $\frac{84}{z} = \frac{7}{12}$

4. Marge drove 220 miles at an average speed of 55 miles per hour. How many hours did it take her to drive that far? _____

• Ratio Problems Involving Totals

A ratio table is useful for organizing information and solving proportions.

If a proportion problem involves a ratio and a total amount, use three rows in the ratio table to organize the information in the problem.

Example: A circus has horses and dogs in a ratio of 2 to 3. If there are 12 horses, what is the total number of dogs and horses in the circus?

	Ratio	Actual Count
horses	2	12
dogs	3	d
total	5	t

Write a proportion. Use the row you know (horses) and the row you want to know (total).

$$\frac{2}{5} = \frac{12}{t}$$

$$2t = 5 \cdot 12$$

$$t = \frac{(5 \cdot 12)}{2}$$

$$t = 30 \text{ The total number of dogs and horses in the circus is 30.}$$

Complete the ratio table to check your answer.

The total number of dogs and horses is 30, so the number of dogs is $30 - 12$, or 18. Therefore, the ratio of horses to dogs is $\frac{12}{18}$, which equals $\frac{2}{3}$. So the answer is correct.

Practice:

- The ratio of men to women in the audience was 8 to 5. If there were 400 men, how many people were in the audience altogether?

- The ratio of mystery books to nonfiction books on a bookcase is 9 to 6. If there are 90 books on the bookcase altogether, how many books are nonfiction?

- A garden center has pine trees and maple trees in a ratio of 3 to 7. If there are 120 trees, how many pine trees does the garden center have?

• Solving Problems Using Scientific Notation

Recall that a number written in **scientific notation** is written as the product of a number equal to or greater than 1 and less than 10, and a power of 10.

- To **multiply** numbers in scientific notation, **multiply the coefficients** to find the coefficient of the product. Then **add the exponents** to multiply the powers of 10.
- To **divide** numbers in scientific notation, **divide the coefficients** to find the coefficient of the quotient. Then **subtract the exponents** to divide the powers of 10.

If the product or quotient is not in scientific form, reposition the decimal point and change the exponent.

Examples:

$$(5.2 \times 10^3)(3.2 \times 10^2) = (5.2 \times 3.2)(10^3 \times 10^2) = 16.64 \times 10^5$$

Since 16.64×10^5 is not in scientific form, rewrite it as 1.664×10^6 .

$$\frac{(1.5 \times 10^2)}{(2.5 \times 10^{-3})} = (1.5 \div 2.5)(10^2 \div 10^{-3}) = 0.6 \times 10^5$$

Since 0.6×10^5 is not in scientific form, rewrite it as 6.0×10^4 .

Notice that when changing a number times a power of 10 to scientific notation, moving the decimal point to the right decreases the exponent. Moving the decimal point to the left increases the exponent.

Practice:

Write each product or quotient in scientific notation.

1. $(2 \times 10^3)(3 \times 10^2) =$ _____

2. $(6 \times 10^4)(6 \times 10^4) =$ _____

3. $\frac{9.9 \times 10^8}{3 \times 10^5} =$ _____

4. $\frac{1.2 \times 10^6}{2 \times 10^3} =$ _____

5. $(2.5 \times 10^3)(4 \times 10^4) =$ _____

6. $\frac{3 \times 10^8}{4 \times 10^2} =$ _____

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• Graphing Functions

A function relates two sets of values.

The first set of values is called the **input**, or the **domain**.

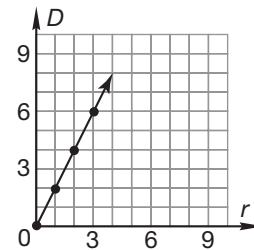
The second set of values is called the **output**, or the **range**.

Each input value is paired with one and only one output value.

We can express a function as a statement, an equation, a table of values, or a graph.

Example: The diameter of a circle is equal to twice its radius.
 This function can also be expressed as an equation: $D = 2r$

Input (r)	Output (D)
0	0
1	2
2	4
3	6



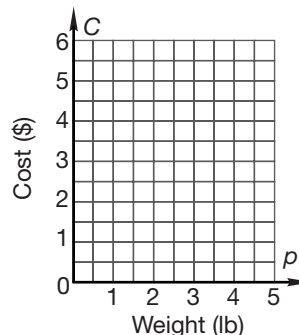
Practice:

1. What are the domain and the range of this function: Apples cost \$0.50 per pound.

2. Write an equation to represent the function relating cost (C) and pounds of apples (p) in Exercise 1.

3. List 5 pairs of input/output values for the function in Exercise 1.

4. Graph the function in Exercise 1.



• Percent of a Whole

A **percent** describes a part of a whole. A percent is also a ratio describing part of 100.

60% means 60 out of 100, or 6 out of 10, or 3 out of 5.

When a problem describes parts of a whole in terms of percent, we can use a three-row ratio table to find the unknown number or percent. The total is 100%.

Example: Twenty-five percent of the cars in the parking lot are black. If 168 cars are not black, what is the total number of cars in the parking lot? Draw a three-row ratio table. Fill in as many parts of the table as you can. Remember that the total percent is 100. So if 25% of the cars are black, then 75% are not black.

	Percent	Actual Count
black	25	b
not black	75	168
total	100	t

Write a proportion using the row you know (not black) and the row you want to know (total).

$$\frac{75}{100} = \frac{168}{t}$$

$$75t = 100 \cdot 168$$

$$t = \frac{(100 \cdot 168)}{75}$$

$$= 224 \quad \text{The total number of cars in the parking lot is 224.}$$

Practice:

Solve each problem by using a proportion and a ratio table.

- Ellen has completed 210 of the puzzles in a book. The book contains 300 puzzles. What percent of the puzzles does Ellen still have left to do?

- Mei has driven 20% of the distance from her office to her home. She is 10 miles away from home. How far away from her home is her office?

- Max has mowed 80% of his lawn. The lawn is 225 yd². How many square yards of the lawn did Max mow?

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Math Course 3, Lesson 49

• Solving Rate Problems with Proportions and Equations

A **rate** is a ratio that relates two quantities. For example, cost relates number of items and price. Speed relates time and distance.

To solve problems about rates, you can use a **proportion**.

Example: Four friends spent \$22 on tickets to a movie. How much would five tickets have cost?

We are given one case and are asked about another case. Draw a ratio box and fill in the sections. Then write a proportion.

	Case 1	Case 2
Cost	\$22	c
Tickets	4	5

$$4c = \$22 \cdot 5$$

$$c = \frac{(\$22 \cdot 5)}{4}$$

$$c = \$27.50 \quad \text{The cost of five tickets would have been } \$27.50.$$

You can also use a **unit rate** to solve problems about rates.

Example: Marilyn drove for 8 hours and traveled 520 miles. How long will it take her to travel 715 miles at the same speed?

First find the unit rate of speed. $\frac{520 \text{ miles}}{8 \text{ hours}} = 65 \text{ miles per hour}$

Then find the time needed to travel 715 miles.

$$\frac{715 \text{ miles}}{65 \text{ miles per hour}} = 11 \text{ hours}$$

It will take Marilyn 11 hours to travel 715 miles at 65 miles per hour.

Practice:

1. If 12 customers are served in 3 hours, how long will it take to serve 20 customers?

2. If 3 rabbits eat $3\frac{3}{4}$ pounds of vegetables each week, how many pounds are needed to feed 5 rabbits in a week? _____

3. A car travels 385 miles in 7 hours. What is its average speed in miles per hour?

4. Fourteen buses can carry 350 students on a field trip. If only 5 buses are available, how many students can go on the trip? _____

• Solving Two-Step Equations

Sometimes it is necessary to use more than one operation to isolate the variable. Choose the operations in the reverse order in which they would be performed in the original equation.

Example: Solve the equation. $3x + 9 = 24$

This equation has two operations: multiplication ($3x$) and addition ($+ 9$). Recall that according to the order of operations, multiplication is done before addition. But to solve the equation, we reverse the order. First undo addition. Then undo multiplication.

$$3x + 9 = 24$$

$$3x + 9 - 9 = 24 - 9 \quad \text{Subtract 9 from both sides to undo addition.}$$

$$3x = 15 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{15}{3} \quad \text{Divide by 3 to undo multiplication.}$$

$$x = 5 \quad \text{Simplify.}$$

$$3(5) + 9 = 24 \quad \text{Check the solution in the original equation.}$$

$$15 + 9 = 24$$

Example: Solve the equation. $\frac{x}{5} - 7 = 28$

This equation has two operations: division ($\frac{x}{5}$) and subtraction (-7). To solve the equation, undo subtraction first. Then undo division.

$$\frac{x}{5} - 7 = 28$$

$$\frac{x}{5} - 7 + 7 = 28 + 7 \quad \text{Add 7 to both sides to undo subtraction.}$$

$$\frac{x}{5} = 35 \quad \text{Simplify.}$$

$$\frac{5x}{5} = 35 \cdot 5 \quad \text{Multiply by 5 on both sides to undo division.}$$

$$x = 175 \quad \text{Simplify.}$$

$$\frac{175}{5} - 7 = 28 \quad \text{Check the solution in the original equation.}$$

$$35 - 7 = 28$$

Practice:

Solve each equation and check each solution.

1. $16w + 32 = 144$

2. $\frac{y}{2} - 8 = 20$

3. $8x - 0.2 = 9.8$

4. $\frac{2}{3}n + \frac{1}{2} = \frac{3}{4}$

5. $\frac{x}{4} - 1.2 = 2$

6. $-3m + 10 = 25$
