• Effect of Scaling on Perimeter, Area, and Volume

If the dimensions of a figure or object are to be changed proportionally, use these ratios between the two figures:

Scale Factor

ratio of perimeters = scale factor
ratio of areas = $(\text{scale factor})^2$
ratio of volumes = $(\text{scale factor})^3$

Example: John has a picture of a room. The picture is 6 inches by 8 inches. He wants to determine what the perimeter of the room would be if it were 10 times the size of the picture. He then wants to determine what the area of the room would be.

Step	Justification
Perimeter of picture = side 1 + side 2 + side 3 + side 4	Given equation
Perimeter of room = 10(side 1) + 10(side 2) + 10(side 3) + 10(side 4)	Multiply by scale factor
Perimeter of room = 10(6) + 10(8) + 10(6) + 10(8)	
10(6 + 8 + 6 + 8) = 280 in.	Distributive property
To find the area of the room:	
Area of picture = $l \times w$ = 6 in. × 8 in.	Given equation
48 in.^{2} $(10)^2$	Multiply by $(\text{scale factor})^2$
48 in.^{2} × 100 = 4800 in.^{2}	
Area of the room = 4800 in.^{2} × 4	

Practice:

1. a. Find the surface area and volume of a 3-inch cube and a 9-inch cube.

   ____

   b. What is the ratio of their surface areas? ______________

   c. What is the ratio of their volumes? ______________

2. If you know the diameters of two circles, how can you find the ratio of their areas?

   ____

Name ____________________

• Areas of Rectangles with Variable Dimensions
• Products of Binomials

An area model can help us visualize multiplying binomials.
Here we illustrate $(x + 4)(x + 3)$.

	x	4
x	x^2	$4x$
3	$3x$	12

We see that there are four products, but $3x$ and $4x$ combine: $x^2 + 7x + 12$.

Use one of two methods to multiply binomials.

1. Use an arithmetic model to find the product.

$$\begin{array}{r} (x + 4) \\ (x + 3) \\ \hline 3x + 12 \\ x^2 + 4x \\ \hline x^2 + 7x + 12 \end{array}$$

2. Use the FOIL order to multiply terms.

Product of two binomials = Products of **F**irst + **O**utside + **I**nside + **L**ast

$$(x + 4)(x + 3) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$$

Be sure to pay attention to the signs when multiplying binomials.

Practice:

Expand:

1. $(x + 2)(x + 7)$ ____________________

2. $(x + 5)(x + 2)$ ____________________

3. $(x - 3)(x + 1)$ ____________________

4. $(x - 2)(x + 2)$ ____________________

5. $(x - 4)(x + 3)$ ____________________

6. $(5 + 3)(5 - 2)$ ____________________

• Equations with Exponents

A **quadratic equation** is an equation containing a variable with an exponent of 2.

$$\boldsymbol{x^2 = 25} \text{ is a } \textbf{quadratic equation.}$$

$$x = 5$$

$$x = -5$$

Every quadratic equation has up to two real solutions.

When solutions are opposite as in the above equation, we can write the equation as:

$$\boldsymbol{x = \pm 5}$$

"$x =$ plus or minus 5"

Example: Solve: $2x^2 + 4 = 36$

Step	**Justification**
$2x^2 = 32$	Subtracted 4 from both sides
$x^2 = 16$	Divided both sides by 2
$x = \pm 4$	Found square roots of 16

Example: Solve: $2x^2 = 36$

$2x^2 = 36$	Given
$x^2 = 18$	Divided both sides by 2
$x = \pm\sqrt{18}$	Found square roots of 18
$x = \pm\sqrt{(3)(3)(2)}$	Simplified
$x = \pm 3\sqrt{2}$	Simplified

Practice:

Solve and check both solutions to each quadratic equation.

1. $2x^2 + 3 = 5$

____

2. $-5x^2 + 10 = -20$

____

3. $\frac{8}{x} = \frac{x}{2}$

____

4. $3x^2 + 3 = 39$

____

5. $\frac{x}{6} = \frac{2}{x}$

____

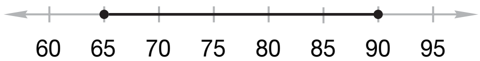
6. $-7x^2 + 5 = -9$

____

Name ______________________________

• Graphing Pairs of Inequalities on a Number Line

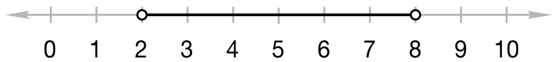
Pairs of inequalities can be graphed on a number line. This graph represents both $h \geq 65$ and $h \leq 90$.



$65 \leq h \leq 90$

The graph is read:
"*h* is greater than or equal to 65 and less than or equal to 90."

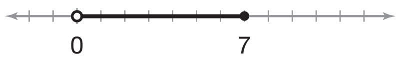
This graph is the **intersection**, or overlap, of the graphs $h \geq 65$ and $h \leq 90$. Another way to write this pair of inequalities is $65 \leq h \leq 90$.
This next graph represents $s > 2$ and $s < 8$.



$2 < s < 8$

This graph is read:
"*s* is greater than 2 and less than 8."

The **open** circles at 2 and 8 indicate that *s* cannot equal 2 or 8.
The next graph represents $t < 40$ or $t > 100$.



$t < 40$ or $t > 100$

This graph is read:
"*t* is less than 40 or greater than 100."

Practice:

Graph these inequalities on a number line.

1. $-5 < x < 6$ ______________________________

2. $2 \leq x < 7$ ______________________________

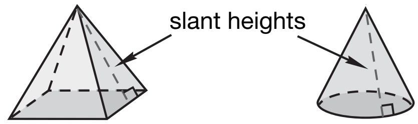
3. $x \leq -6$ or $x \geq 6$ ______________________________

4. $x < 1$ or $x > 3$ ______________________________

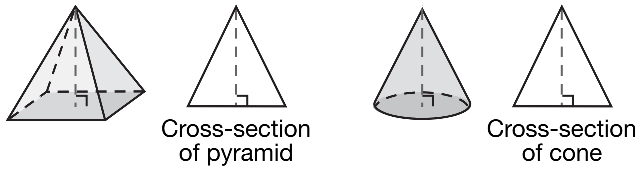
5. $0 < x \leq 7$ ______________________________

• Slant Heights of Pyramids and Cones

- The **slant height** of a pyramid or cone is the diagonal distance along the surface from the apex to the base.

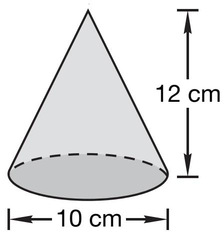


We can find the slant height of a solid by examining a cross-section: Look at the vertical **cross-section** of a pyramid or cone at the apex. The cross-section is an **isosceles triangle**:



The height divides the cross-section into two right triangles. So, the slant height is equivalent to the hypotenuse of one of the right triangles.
If we know the dimensions of the base and the height of a solid, then we know the measures of two legs of the right triangles. Use the Pythagorean Theorem to find the hypotenuse which is the slant height of the solid.

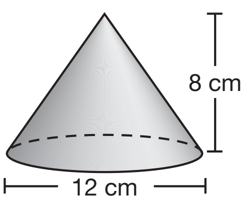
Example:

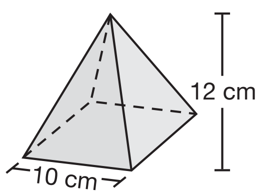


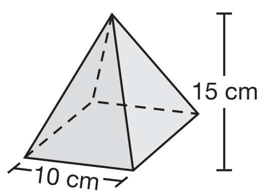
Step 1:	Determine the length of the legs of the right triangle formed by the cross-section.	Radius = 5 cm Height = 12
Step 2:	Apply the Pythagorean Theorem	$a^2 + b^2 = c^2$
Step 3:	Solve	$5^2 + 12^2 = c^2$ $169 = c^2$
	The slant height is 13 cm.	$13 = c$

Practice:

1. Find the slant height of the following figures. Leave irrational numbers in radical form.

a. 

b. 

c. 

________________ ________________ ________________

2. Compared to the height of a cone, the slant height of a cone is ________

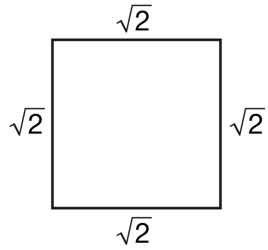
A. shorter **B.** the same **C.** longer

• Geometric Measures with Radicals

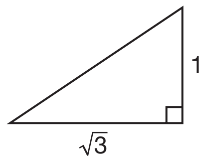
- To add radicals, group common radicals with a coefficient:

$$\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 4\sqrt{2}$$

The perimeter of this square is $4\sqrt{2}$



To find the perimeter of this triangle, first use the Pythagorean Theorem.



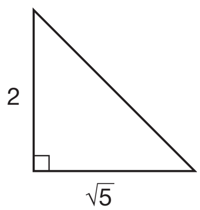
Step	**Justification**
$a^2 + b^2 = c^2$	Pythagorean Theorem
$1^2 + (\sqrt{3})^2 = c^2$	Substituted for *a* and *b*
$1 + 3 = c^2$	Simplified
$4 = c^2$	Simplified
$2 = c$	Found positive square root of 4.

Add the three sides of the triangle to find the perimeter.

$$\text{Perimeter} = 1 + 2 + \sqrt{3}$$
$$P = 3 + \sqrt{3}$$

Practice:

1. Find the perimeter of the triangle.



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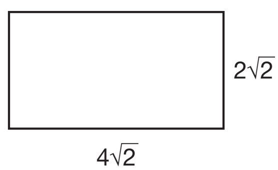
2. Find the perimeter and the area of the rectangle.



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3. Find the perimeter and area of a square with vertices at (3, 0) (0, 3) (–3, 0) (0, –3).

____

• Recursive Rules for Sequences

- A **sequence** is an ordered list of numbers that follow a certain pattern or rule.

$$\{3, 6, 9, 12, 15, \ldots\}$$

We can distinguish between the **position** of the term (such as 1st, 2nd, 3rd) and the **value** of a term (such as 2, 4, 8).

Term position (n)	1	2	3	4	. . .
Term value (a)	3	6	9	12	. . .

The formula $\boldsymbol{a_n = 3n}$ will generate any term of the above sequence.
To generate the term in the 8th position, substitute 8 for n in the formula:

$$\boldsymbol{a_8 = 3(8) = 24}$$

Remember, the subscripts only indicate position; they are not involved in an calculations.

- **Recursive Formulas**

We can also write formulas that show how a term in a sequence relates to a preceding term or terms. This kind of formula is called a **recursive formula**.

$$\begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 3 \end{cases}$$

This **recursive formula** tells us that in the above sequence, the value of a term is 3 more than the preceding term. We indicate the first term (a_1) of the sequence. In this case the first term is three.

Practice:

1. Write a recursive formula for the following sequence: ____________________

1, 5, 25, 125, . . .

2. Write a recursive formula for the following sequence: ____________________

2, 4, 6, 8, . . .

3. Which formula below generates the terms of the following sequence? _________

2, 5, 11, 23, . . .

A. $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 5 \end{cases}$ **B.** $\begin{cases} a_1 = 2 \\ a_n = 2a_{n-1} + 1 \end{cases}$ **C.** $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 1 \end{cases}$

4. The terms of the following sequence are generated with the formula $a_n = 4n - 1$. Write a recursive formula for this sequence:

3, 7, 11, 15, . . .

____

• Relations and Functions

• A **relation** is a pairing of two sets of information.
Relations may be described in several ways:

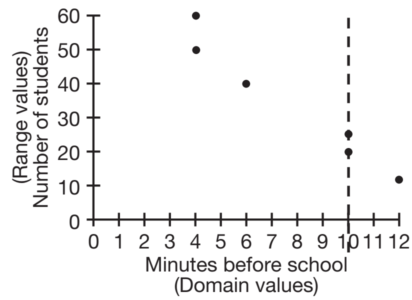
1. set notation **2.** with a table of pairs **3.** with an equation or graph

> A function is a relation, but not all relations are functions.

A function takes **one input** value and assigns it exactly **one output value**. If a relation has more than one output for an input, it is not a function.

Example: This graph represents the number of minutes Preston arrives before school starts and the number of students at school at that time:

m	*n*
4	50
10	20
12	12
10	25
6	40
4	60



Notice that some points share the same input value. The vertical line through the points indicates that the relation is not a function.

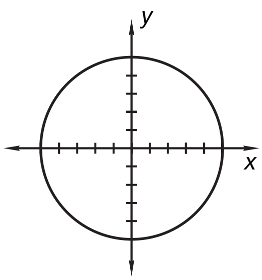
Vertical Line Test

> If a vertical line can be drawn that passes through more than one point on the graph of a relation, then the relation is not a function.

Practice: State whether each graph or table is a function.

1.

x	*y*
–5	0
0	5
5	0
0	–5



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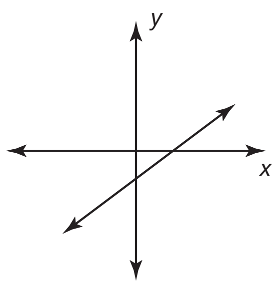
2.



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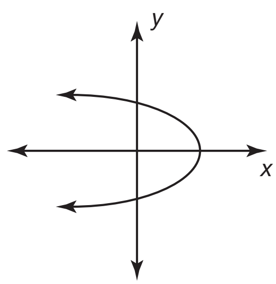
3.



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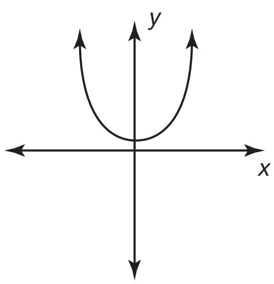
4.



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5.

x	*y*
4	6
3	7
2	8
1	9
0	10

____

6.

x	*y*
2	1
3	2
4	3
3	4
2	5

____

• Inverse Variation

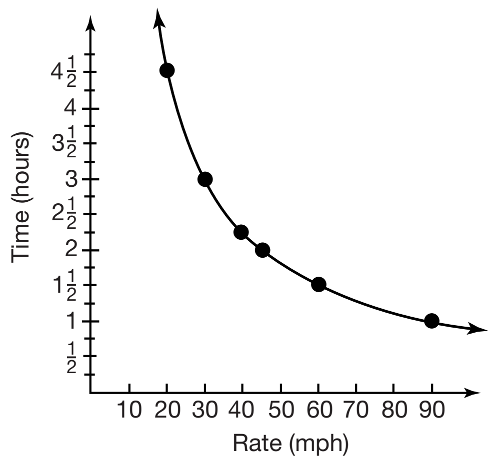
Direct Variation	Inverse Variation
$\frac{y}{x} = k$	$xy = k$
The quotient of two variables is a constant.	The product of two variables is a constant.

- Here's an example of an **inverse variation**:

Heidi travels 90 miles to visit her grandmother. If she averages 60 miles per hour, the trip takes $1\frac{1}{2}$ hours. However, sometimes, while traveling, Heidi makes a few stops, so her average speed varies. This affects the travel time, but the distance stays constant.

Average Rate (mph)	Travel Time (hr)	Distance
60	$1\frac{1}{2}$	90
40	$2\frac{1}{4}$	90
30	3	90

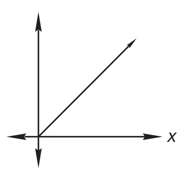
More rate and time values are graphed to the right:
This graph is characteristic of inverse variation.
The function is non-linear.



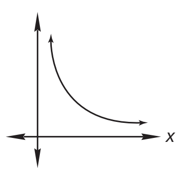
Practice:

1. Describe each equation or graph in a–d as inverse variation or direct variation.

a. $\frac{y}{x} = k$ **b.** $x \cdot y = k$ **c.**



d.



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2. Determine if the variables in tables a and b are inversely proportional and explain how you made that determination.

a.

x	y
2	4
3	1
4	3

____
____
____

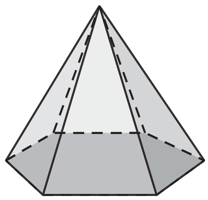
b.

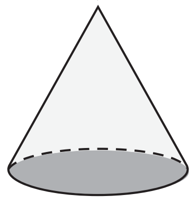
x	y
2	4
1	8
4	2

____
____
____

• Surface Areas of Right Pyramids and Cones

Formula for the Lateral Surface Area of a Right Pyramid	**Formula for the Lateral Surface Area of a Right Cone**
$A_s = \frac{ps}{2}$ or $\frac{1}{2}ps$	$A_s = \pi rs$
A_s is lateral surface area	A_s is lateral surface area
p is perimeter of the base	r is radius of the base
s is slant height	s is slant height





To find the lateral surface area of a right pyramid, you can find the area of one face and multiply by the number of sides, or you can apply the formula from above.

Practice:

1. Find the lateral surface area of a square pyramid with base sides of 4 inches long and a slant height of 5 inches?

____

2. What is the total surface area of the square pyramid described in Exercise 1?

____

3. Find the lateral surface area of a cone with a diameter of 10 cm and a slant height of 16 cm. Express your answer in terms of π.

____

4. Find the lateral surface area of a right pyramid with a slant height of 10 cm and a regular octagonal base with sides 6 cm long.

____